Journal of Computational and Applied Mathematics 257 (2014) 212–239

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Singular optimal dividend control for the regime-switching Cramér–Lundberg model with credit and debit interest

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HIGHLIGHTS

- We study a regime-switching compound Poisson model with credit and debit interest.
- The surplus process is controlled by subtracting the cumulative dividends.
- Our objective is to find an optimal dividend strategy.
- We show that a regime-switching band strategy is optimal.

ARTICLE INFO

Article history:
Received 31 January 2013
Received in revised form 15 August 2013

Keywords:
Band strategy
Compound Poisson process
Cramér–Lundberg model
Dividend optimization
Regime switching
Singular control

ABSTRACT

We investigate the dividend optimization problem for a company whose surplus process is modeled by a regime-switching compound Poisson model with credit and debit interest. The surplus process is controlled by subtracting the cumulative dividends. The performance of a dividend distribution strategy which determines the timing and amount of dividend payments, is measured by the expectation of the total discounted dividends until ruin. The objective is to identify an optimal dividend strategy which attains the maximum performance. We show that a regime-switching band strategy is optimal.

1. Introduction

The problem of finding the optimal strategy which determines when to pay out dividends and how much to pay out is an important topic in finance and actuarial science [1]. Most works on the dividend optimization problem are based on either the diffusion setting or the compound Poisson (Cramér–Lundberg) setting. The former setting is an approximation of the latter and has better mathematical tractability. There is a wealth of work studying the dividend optimization problem and its extensions under the diffusion setting (see [2–7] and the references therein). The classical compound Poisson model is more directly appealing for insurance modeling. Under such a setting, the dividend optimization along with optimal reinsurance problem was solved in [1]. Albrecher and Thonhauser [8] studied the optimization problem for the compound Poisson model by adding constant force of interest. Kulenko and Schmidli [9] considered the optimization problem for the compound Poisson model with capital injections. Most recently, [10] investigated the problem by allowing the insurance company to continue its business when the surplus is negative and above a critical level through refinancing and [11] solved the optimal dividend problem for the case of bounded dividend rates.

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http://dx.doi.org/10.1016/j.cam.2013.08.033

The majority of the papers in the literature of dividend optimization consider only one source of uncertainty in modeling the evolution of the cash surplus, which is a Brownian motion in the diffusion setting and a compound Poisson arrival process in the Cramér–Lundberg setting. However, empirical studies showed that a company's earnings are also affected by the external environment regime, e.g. macroeconomic conditions (see [12] and the references therein for details). Hence, models with regime switching are more realistic. A substantial literature in econometrics supports that a finite-state Markov process is appropriate to model the external environment regime (see [13] and references therein). Markov regime-switching models have been used in different contexts including consumption–investment (see, for example, [14]), portfolio management (see, for example, [15]), option pricing (see, for example, [16,17]), risk theory (see, for example, [18]) etc. Recently, research devoted to studying the optimal dividend problem for models with regime switching has appeared. Sotomayor and Cadenillas [12] studied a diffusion process with two regimes and solved the optimal dividend problem for both the case of bounded dividend payment rates and unbounded dividend rates. Jiang and Pistorius [13] solved the dividend optimization problem for a regime-switching diffusion process with multiple regimes. Wei et al. [19] studied the compound Poisson model with regime switching and showed that the value function for the impulse control problem is the unique viscosity solution of the corresponding quasi-variational inequality. Yin et al. [20] studied a limit system of the compound Poisson model with regime switching and showed that the optimal dividend strategy for the limit system is asymptotically optimal for the original model. However, for the regime-switching compound Poisson model, no optimal strategies have been identified for either the singular or the classical control problem.

In this paper, we consider the singular dividend optimization problem for the compound Poisson model with Markov regime switching. Our goal is to find an optimal dividend strategy among a set of admissible strategies with unrestricted dividend rates so that the total expected discounted dividends are maximized. The inclusion of regime switching makes it hard to directly apply the viscosity approach used for the model with no regimes. Instead, we solve the problem by first studying an auxiliary optimization problem with a different optimization criterion where only the dividends up to the first regime switch plus a terminal random value are included in the performance functional. By building a connection between the original and the auxiliary optimization problem, we use the optimization results obtained for the latter to derive results for the original problem. We find that a strategy of a band type is optimal. Our results show the optimal strategy is stationary and depends on the level of surplus and the environment regime at the time.

This paper is organized as follows. In Section 2 we present the problem. In Section 3 we study an auxiliary optimization problem with a different performance functional and present the optimal solution for this new problem. In Section 4, we construct an optimal dividend strategy with the assistance of the optimization results for the auxiliary problem. A conclusion is provided in Section 5.

2. Problem formulation

Consider a company operating under an environment described by the finite state stochastic process $\{J_t;\, t \geq 0\}$. When the environment status (regime) is i, claims arrive according to a Poisson process with intensity rate λ_i and premiums are collected continuously at rate p_i. Let S_k denote the arrival time of the kth claim and U_k the size of this claim. Claim sizes, conditioning on the regimes of the arrival times of these claims, are independent and independent of the claim arrival process. And given $J_{S_k} = i$, the random variable U_k follows the distribution $F_i(\cdot)$. Let $N(t)$ denote the number of claims up to time t. Then, $N(t) = \#\{k : S_k \leq t\}$. Assume that the insurance company, when in regime i, earns interest under a constant force $r_i\ (> 0)$ for its positive surplus (reserve), and, if the surplus drops below zero, could borrow money with the amount equal to the deficit. The company will repay the debt and the interest charged at a force α_i continuously at the same rate as the incoming premium rate.

For any x, define $(x)^+ = \max\{x, 0\}$ and $(x)^- = -\min\{x, 0\}$. Let R_t denote the surplus at time t. Then the surplus process $\{R_t;\, t \geq 0\}$ follows the following dynamics:

$$\mathrm{d}R_t = (p_{J_{t-}} + r_{J_{t-}}(R_{t-})^+ - \alpha_{J_{t-}}(R_{t-})^-)\mathrm{d}t - \mathrm{d}\left(\sum_{k=1}^{N(t)} U_k\right). \tag{2.1}$$

Assume that all the above random quantities are defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. The environment process $\{J_t;\, t \geq 0\}$ is a Markov chain with the state space $\mathbb{E} = \{1, 2, \ldots, \kappa\}$ and the transition intensity matrix $Q = (q_{ij})_{\kappa\times\kappa}$. We define $q_i = -q_{ii} = \sum_{j\neq i} q_{ij}$ for $i \in \mathbb{E}$.

We introduce the notations $P_{(x,i)}(\cdot) = P(\cdot|R_0 = x, J_0 = i)$ and $E_{(x,i)}[\cdot] = E[\cdot|R_0 = x, J_0 = i]$.

Suppose the company will pay out dividends to the shareholders from surplus. We use L_t to denote the cumulative dividends paid up to time t and call the stochastic process $L = \{L_t;\, t \geq 0\}$ a dividend strategy. Then, it is fair to set $L_0 = 0$ and let L be non-decreasing in t. The dividend payment decision at any time will be made based on past information only and not on any future information, and therefore the amount of dividend to be paid immediately after t depends on the information up to time t. As a result, it is reasonable to assume that the stochastic process L is predictable. It is natural to assume that L is also left continuous with limits from the right side (cáglád). Since the process L is nondecreasing and left continuous, it has the following decomposition: $L_t = L_t^c + \sum_{0\leq s<t}(L_{s+} - L_s)$, where $\{L_t^c;\, t \geq 0\}$ is the continuous part of the process $\{L_t;\, t \geq 0\}$. Furthermore, $L_{t+} = L_t^c + \sum_{0\leq s\leq t}(L_{s+} - L_s)$.