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# A note on the dynamic analysis using the generalized finite difference method

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ABSTRACT

This paper shows the application of the generalized finite difference method (GFDM) to the problem of dynamic analysis of beams and plates. The stability conditions for a fully explicit algorithm are given for beams and plates. Measures of the irregularity of the clouds of points for beams and plates are given. Various cases of vibrations of beams and plates have been solved and the results show the accuracy of the method for irregular clouds of nodes.



## 1. Introduction

The generalized finite difference method (GFDM) is evolved from the classical finite difference method (FDM). GFDM can be applied over general or irregular clouds of points [1]. The basic idea is to use moving least squares (MLS) approximation to obtain explicit difference formulae which can be included in the partial differential equations [2]. Benito, Ureña and Gavete have made interesting contributions to the development of this method [3–8]. The paper [9] shows the application of the GFDM in solving parabolic and hyperbolic equations.

This paper describes how the GFDM can be applied for solving dynamic analysis problems of beams and plates using an explicit scheme [10–13].

The paper is organized as follows. Section 1 is the introduction. Section 2 describes the explicit generalized finite difference schemes, Section 2.1 the explicit GFDM scheme for beams and Section 2.2 the explicit GFDM scheme for plates. Section 3 studies the convergence: consistency and the von Neumann stability, Section 3.1.1 the truncation error and consistency for beams, and Section 3.1.2 the truncation error and consistency for plates. Section 3.2.1 studies the von Neumann stability for beams and Section 3.2.2 the von Neumann stability for plates. Section 4 analyses the relation between stability and irregularity of a cloud of nodes. Section 4.1 defines the index of irregularity of a cloud of nodes for beams and Section 4.2 the index of irregularity of a cloud of nodes for plates. In Section 5 some applications of the GFDM for solving

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problems of dynamic analysis of beams are included. In Section 6, some applications of the GFDM for solving problems of dynamic analysis of plates are included. Finally, in Section 7 some conclusions are given.

## 2. Explicit generalized finite difference schemes

### 2.1. Vibrations of beams

First, we consider the cases of vibrations of beams.

Let us consider the problem governed by the following partial differential equation (pde)

$$\frac{\partial^2 U(x,t)}{\partial t^2} + C_1^2 \frac{\partial^4 U(x,t)}{\partial x^4} = F_1(x,t) \quad x \in (0,L),\ t > 0 \tag{1}$$

with boundary conditions at the ends of the beam of length $L$ for each particular case and initial conditions

$$U(x,0) = 0; \qquad \left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = F_2(x) \tag{2}$$

where $F_1$ and $F_2$ are two known smooth functions, the constant $C_1$ depends on the material and geometry of the beam.

We use the explicit difference formulae for the values of partial derivatives in the space variable. The intention is to obtain explicit linear expressions for the approximation of partial derivatives in the points of the domain.

First of all, an irregular grid or cloud of points is generated in the domain. On defining the composition central node with a set of $N$ points surrounding it (henceforth referred as nodes), the star then refers to the group of established nodes in relation to a central node. Each node in the domain have an associated star assigned [3,2,4,1].

If $u_0$ is an approximation of fourth-order for the value of the function at the central node ($U_0$) of the star, with coordinate $x_0$ and $u_j$ is an approximation of fourth-order for the value of the function at the rest of nodes, of coordinates $x_j$ with $j = 1, \ldots, N$, then, according to the Taylor series expansion

$$U_j = U_0 + h_j \frac{\partial U_0}{\partial x} + \frac{h_j^2}{2}\frac{\partial^2 U_0}{\partial x^2} + \frac{h_j^3}{6}\frac{\partial^3 U_0}{\partial x^3} + \frac{h_j^4}{24}\frac{\partial^4 U_0}{\partial x^4} + \cdots \tag{3}$$

where $h_j = x_j - x_0$.

If in Eq. (3) the terms over fourth order are ignored, it is then possible to define the function $B_4(u)$ as in [3,2,4,9,6,1,8]

$$B_4(u) = \sum_{j=1}^{N} \left[ \left( u_0 - u_j + h_j \frac{\partial u_0}{\partial x} + \frac{h_j^2}{2}\frac{\partial^2 u_0}{\partial x^2} + \frac{h_j^3}{6}\frac{\partial^3 u_0}{\partial x^3} + \frac{h_j^4}{24}\frac{\partial^4 u_0}{\partial x^4} \right) w(h_j) \right]^2 \tag{4}$$

where $w(h_j)$ is the denominated weighting function.

If the norm (4) is minimized with respect to the partial derivatives the linear equations system is obtained

$$\boldsymbol{A_4 D_{u_4}} = \boldsymbol{b_4} \tag{5}$$

where

$$\boldsymbol{A_4} = \begin{pmatrix} \sum_{j=1}^{N} h_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_i^4}{6} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{24} w^2 \\ & \sum_{j=1}^{N} \frac{h_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{12} w^2 & \sum_{j=1}^{N} \frac{h_j^6}{48} w^2 \\ & & \sum_{j=1}^{N} \frac{h_j^6}{36} w^2 & \sum_{j=1}^{N} \frac{h_j^7}{144} w^2 \\ & SYM & & \sum_{j=1}^{N} \frac{h_j^8}{576} w^2 \end{pmatrix} \tag{6}$$

and

$$\boldsymbol{D_{u_4}} = \left\{ \frac{\partial u_0}{\partial x} \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^3 u_0}{\partial x^3} \quad \frac{\partial^4 u_0}{\partial x^4} \right\}^T \tag{7}$$