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Numerical inverse Lévy measure method for infinite shot noise series representation[☆]

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ABSTRACT

Infinitely divisible random vectors without Gaussian component admit representations with shot noise series. We analyze four known methods of deriving kernels of the series and reveal the superiority of the inverse Lévy measure method over the other three methods for simulation use. We propose a numerical approach to the inverse Lévy measure method, which in most cases provides no explicit kernel. We also propose to apply the quasi-Monte Carlo procedure to the inverse Lévy measure method to enhance the numerical efficiency. It is known that the efficiency of the quasi-Monte Carlo could be enhanced by sensible alignment of low discrepancy sequence. In this paper we apply this idea to exponential interarrival times in the shot noise series representation. The proposed method paves the way for simulation use of shot noise series representation for any infinite Lévy measure and enables one to simulate entire approximate trajectory of stochastic differential equations with jumps based on infinite shot noise series representation. Although implementation of the proposed method requires a small amount of initial work, it is applicable to general Lévy measures and has the potential to yield substantial improvements in simulation time and estimator efficiency. Numerical results are provided to support our theoretical analysis and confirm the effectiveness of the proposed method for practical use.

1. Introduction

The infinitely divisible law consists of three independent components: a constant, a centered normal random vector and a Poisson jump component. The Poisson component is governed in full by the so-called Lévy measure. If Lévy measure is infinite, then its Poisson component consists of an infinite number of jumps. Obviously, it is impossible to generate infinitely many jumps in practice. Possible simulation methods can be summarized as follows.

(i) Direct generation: when an explicit density function, or an alternative exact simulation method, is available, a direct sample generation, even with further improvement for Monte Carlo methods, is straightforward in principle. Stable, exponential, gamma, geometric and negative binomial are in this category. To simulate random variables from complex distributions, but still with density known in the closed form, numerical inversion methods of a distribution function have been applied to generalized hyperbolic, normal inverse Gaussian and Meixner distributions. See, for example, [1–5].

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(ii) Generation from compound Poisson: if a neighborhood of the origin of the Lévy measure is discarded or replaced by its mean value, then the remainder term is a compound Poisson random vector with a constant shift. It converges to its true random vector, as the intensity of the discarded part of the Lévy measure decreases. However, when the discarded part of the Lévy measure is too intense, the component may produce a substantial error. The discarded component can be approximated by a Gaussian random vector with an appropriate small variance under mild conditions on the Lévy measure [6,7]. This approximation may complement a method through series representations when the series converges slowly.
(iii) Generation with shot noise series representation: infinite shot noise series representations provide a simple simulation method of infinitely divisible random vectors and associated stochastic differential equations. A disadvantage of this method is that some series may converge at an extremely slow rate. One might need a huge number of terms to reach a desired accuracy of the approximation. With ever increasing computational speed, a slow convergence may no longer be a serious issue of practical importance in some applications. Also, it is sometimes the case that the representation is intricate with many different random sequences involved. See, for example, [8–11] for simulation use of infinite series representations.

Series representations involving Poisson arrival times are given by Ferguson and Klass [12], for real independent increment processes without Gaussian component and with positive jumps. Those are extended by LePage [13] to multidimensional settings. The uniform convergence of those representations is established in [14], while they are related to the Lévy–Ito decomposition of processes with independent increments in [15]. The simulation of nonnegative infinitely divisible random variables is investigated and their series representations as a special form of generalized shot noise is developed in [16]. The same approach is used in [17] as a general pattern for series representations of Banach space valued infinitely divisible random vectors.

Among several different methods of deriving a kernel of shot noise series representation, the inverse Lévy measure method [12,13] is most attractive from a simulation point of view for the reason that, in principle, we only need to generate Poisson arrival times for jump size and a few others for jump timing and direction.

In this paper, we rigorously validate the inverse Lévy measure method in computation. To this end, in Section 3, we analyze and compare four known methods of deriving kernels of shot noise series, that is, the inverse Lévy measure method [12,13], the rejection method [17], the thinning method [17] and Bondesson's method [16]. In particular, we rigorously prove in Theorem 3.1 that the inverse Lévy measure method simulates more mass of Lévy measure tails than the other three methods, under a common finite truncation.

It is however often the case that no kernel is available in the closed form through the inverse Lévy measure method. Derflinger et al. [18] propose a numerical inversion method for generating random variates from continuous distributions when only the density function is given. Their algorithm is based on Newton interpolation of the inverse CDF and Gauss–Lobatto integration. The method is applicable to calculate values for inverse of the Lévy measure although in our case the Lévy measure is not normalized. Motivated by their work [18], we propose a model-free efficient numerical procedure to simulate infinitely divisible random distributions by directly applying the inverse Lévy measure method and discuss its advantages and limitations. It is worth emphasizing that although a naive direct method is always available and can be easily performed by solving a nonlinear equation to get a sample from a Lévy measure with the Newton algorithm, it is far less efficient than the proposed method, especially when using in the Monte Carlo framework that involves many iteration procedures. Furthermore, the proposed method is applicable to a broad range of problems as long as their Lévy measures are available in the closed form. In order to further enhance the efficiency of the numerical approach we discuss in Section 5 the applicability of the quasi-Monte Carlo (QMC) method. The QMC method has been proposed as a competitive alternative for Monte Carlo (MC) method. By relying on a specially constructed sequence known as the low discrepancy sequence, the QMC achieves a convergence rate of $O(N^{-1}\log^d N)$, in dimension d and sample size N, known as the Koksma–Hlawka bound. This means, at least asymptotically, the convergence rate is far more superior than the classical MC rate of $O(N^{-1/2})$. See a monograph of Niederreiter [19] for a detailed discussion of the low-discrepancy sequence. Many authors report that the QMC yields a much higher accuracy than the MC method, especially in financial applications.

From a practical point of view, on the other hand, it is known that the success of QMC is intricately related to the notion of effective dimension, originally discussed by Caflisch et al. [20] who introduced this notion using the analysis of variance decomposition of a function. It can be shown that the superior rate of QMC could be attained when the effective dimension of the integrand is small. In this paper we propose to employ the QMC method combined with the numerical inverse Lévy method. In the same spirit of [8], the multi-dimensional low discrepancy sequence is carefully assigned to a set of random variables to decrease the effective dimension. Section 6 presents numerical results to illustrate the superiority of our numerical inverse Lévy measure method over the three other methods for simulation use and the applicability of low discrepancy sequences. Finally, Section 7 concludes and outlines directions for further research. To maintain the flow of the paper, we collect proofs in the Appendix. To avoid overloading the paper with rather lengthy proofs of somewhat routine nature, we omit non-essential details in some instances.

2. Shot noise series representation of Lévy processes

Let us begin this section with the notations which will be used throughout the paper. We denote by $\mathbb{R}^d$ the d-dimensional Euclidean space with the norm $\|\cdot\|$, $\mathbb{R}_0^d := \mathbb{R}^d \setminus \{0\}$, $\mathbb{R}_+ := (0, +\infty)$ and $\mathscr{B}(\mathbb{R}_0^d)$ is the Borel σ-field of $\mathbb{R}_0^d$. We let $\mathbb{N}$ be