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A lattice method for option pricing with two underlying assets in the regime-switching model

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ABSTRACT

In this work we develop an efficient lattice approach for option pricing with two underlying assets whose prices are governed by regime-switching models. Jump amplitudes are specified in a way such that the lattice achieves complete node recombination along each asset variable and grows quadratically as the number of time steps increases. Jump probabilities are obtained by solving a related quadratic programming problem. The weak convergence of the discrete lattice approximations to the continuous-time regime-switching diffusion processes is established. The lattice is employed to price both European and American options written on the maximum and minimum of two assets in different regimes. Numerical results are provided and compared for the European options with a Monte-Carlo simulation approach.

1. Introduction

Lattice has been an important approach in computational finance ever since the innovative development of the binomial tree by Cox, Ross and Rubinstein [1] for pricing options on a single asset. A number of papers have extended the CRR binomial tree method to multiple assets and proposed lattice methods for pricing multivariate contingent claims. See [2–4], among others. Note that in these papers, the asset price is assumed to follow a constant geometric Brownian motion (GBM) model and the jump amplitudes of the trees remain constant as the time evolves.

Regime-switching models have drawn considerable attention in recent years in financial mathematics and computational finance, due to their capability of modeling non-constant and perhaps random market parameters (e.g. volatility and interest rate) and their comparably inexpensive computation. In this setup, asset prices are dictated by a number of stochastic differential equations coupled by a finite-state Markov chain, which represents randomly changing economical factors. Model parameters (drift and volatility coefficients) are assumed to depend on the Markov chain and are allowed to take different values in different regimes. As a result, both continuous dynamics and discrete events are present in the regime-switching model. Regime-switching models have been used in various subjects in financial mathematics including equity options [5–18], bond prices and interest rate derivatives [19–21], energy and commodity derivatives [22,21,23], portfolio selection [24], trading rules [25–29], and others. We refer the reader to Yin and Zhu [30] on the recent studies of regime-switching systems.

Developments of lattice approaches for valuing options on a single asset using regime-switching models have been presented in a number of articles. See [5,6,10,14,21,17,18], among others. Aingworth et al. [5] and Guo [10] extend the

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CRR binomial tree [1] in a straightforward way to a regime-switching geometric Brownian motion (GBM) model, resulting in a partially recombining tree. One disadvantage of their tree constructions is that the number of nodes can grow too fast as the number of time steps increases. According to Nelson and Ramaswamy [31], a lattice approximation to a (one-dimensional) diffusion is "computationally simple" if the number of nodes grows at most linearly in the number of time steps. To reduce the growth rate of nodes, Bollen [6] considers a GBM model with two regimes and proposes a pentanomial tree that achieves complete node recombination and grows linearly. Liu [14] adapts a similar idea of [6] and develops a linear tree for the regime-switching GBM model allowing any number of regimes. It is numerically demonstrated in [14] that the new recombining tree is efficient and accurate in pricing both European and American options with a single asset. Liu [21] extends the tree method [14] to a class of regime-switching mean-reverting models that have been frequently used for stochastic interest rates, energy and commodity prices. Yuen and Yang [17,18] develop a trinomial tree method for option pricing in Markov regime-switching models and use the proposed tree to price Asian options and equity-indexed annuities.

In this work we extend the approach of [14] to options with two underlying assets whose prices are governed by the regime-switching GBM models. In constructing the lattice, jump amplitudes for the two variables are specified in a way such that the lattice achieves complete node recombination along each variable and as a result, the two-dimensional lattice grows quadratically as the number of time steps increases. Jump probabilities are determined by solving a related quadratic programming problem. In addition to the algorithm, by following a method of martingale problem formulation, we establish the weak convergence of the discrete lattice approximations to the continuous-time regime-switching GBM processes. We employ the new lattice method to price both European and American options written on the maximum and minimum of two assets in different regimes and report numerical results. For the European options, we also numerically compare our lattice method with a Monte-Carlo simulation approach.

The paper is organized as follows. Section 2 is devoted to the lattice development. The regime-switching model is introduced first. The details of constructing the lattice are given next. Section 3 is concerned with the weak convergence of the lattice approximations to the continuous-time processes. Section 4 is concerned with the numerical study. The lattice approach is employed to price both European and American options written on the maximum and minimum of two assets in different regimes. For the European options, the results are compared with a Monte-Carlo simulation approach. Section 5 provides further remarks and concludes the paper.

2. Regime-switching lattice with two assets

We consider a continuous-time Markov chain α_t that takes a value at a time among m_0 different states, where m_0 is the total number of states considered for the model. Each state represents a particular regime and is labeled by an integer i between 1 and m_0. Hence the state space of α_t is the set $\mathcal{M} := \{1, \dots, m_0\}$. For example, if $m_0 = 2$ (two regimes), then $\alpha_t = 1$ may indicate a bullish market and $\alpha_t = 2$ a bearish market. In this paper we assume that the Markov chain α_t is observable.

We note that introducing a Markov chain into the model results in an incomplete market [32,10]. It has been known that the absence of arbitrage opportunities is equivalent to the existence of an equivalent Martingale measure. However, the equivalent Martingale measure is not unique if the market is incomplete. Elliott, Chan and Siu [32] and Siu [33] employ a regime-switching random Esscher transform to choose an equivalent Martingale measure for pricing derivatives. We decide not to include the argument in this paper and instead assume that the risk-neutral probability space $(\Omega, \mathcal{F}, \widetilde{\mathcal{P}})$ is given. Another related and interesting issue is whether one should price the regime-switching risk. We refer the readers to Elliott, Siu and Badescu [34], Siu [35] and Siu and Yang [36] for more discussions.

In this work we consider options written on two correlated underlying risky assets whose prices $S_1(t)$ and $S_2(t)$ are given by the following regime-switching geometric Brownian motions. For $i = 1, 2$,

$$\frac{dS_i(t)}{S_i(t)} = r(\alpha_t)dt + \sigma_i(\alpha_t)dB_i(t), \quad t \geq 0, \tag{2.1}$$

where $\sigma_i(\alpha_t)$ is the volatility rate of the ith asset, $r(\alpha_t)$ is the risk-free interest rate, $B_1(t)$ and $B_2(t)$ are two correlated Brownian motions with $dB_1(t)dB_2(t) = \rho dt$ where $-1 < \rho < 1$ is the correlation coefficient between the two Brownian motions. We assume that $B_1(t)$ and $B_2(t)$ are independent of the Markov chain α_t. Note that $\sigma_i(\alpha_t)$ and $r(\alpha_t)$ depend on the Markov chain α_t, indicating that they can take different values in different regimes. We assume that $\sigma_i(\alpha) > 0$, $\alpha = 1, \dots, m_0, i = 1, 2$.

Remark 1. We could assume that the correlation coefficient ρ depends on the market regime as well in our lattice model. However, noting that (2.1) implies $\frac{dS_1(t)}{S_1(t)}\frac{dS_2(t)}{S_2(t)} = \sigma_1(\alpha_t)\sigma_2(\alpha_t)\rho dt$, we see that the regime-dependent feature for the correlation between the two risky asset returns is captured by assuming that the volatility rates depend on the market regime. For this reason we choose to use a constant correlation coefficient ρ in the exposition of our lattice construction.

To proceed, let $X_i(t) = \ln(S_i(t)/S_i(0))$. Then $S_i(t) = S_i(0)e^{X_i(t)}$ where $X_i(t)$, $i = 1, 2$ are the solutions of the stochastic differential equations:

$$dX_i(t) = a_i(\alpha_t)dt + \sigma_i(\alpha_t)dB_i(t), \quad X_i(0) = 0, \tag{2.2}$$