Journal of Computational and Applied Mathematics 240 (2013) 87–98

Contents lists available at SciVerse ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Multiresolution terrain modeling using level curve information

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ARTICLE INFO

Article history:
Received 1 February 2012
Received in revised form 9 July 2012

Keywords:
Multiresolution
Terrain compression
Level curves
Digital terrain model
Contour interpolation

ABSTRACT

The core of the presented multiresolution method is an algorithm for removing recursively the level curves according to some error criterion. This allows us to obtain a sequence of approximations of the terrain where the difference between two consecutive approximations is only one curve: the less "important". In other words, the input curves are sorted in such a way that the n most relevant curves are contained in the n-th resolution level. For a given curve, the relevance criterion is the error computed using a function interpolating the remaining curves. Hence, to fully formulate the multiresolution algorithm a function interpolating the contour lines of the terrain is necessary. We note that a contour line representation has a higher density in the horizontal direction than in the vertical. To alleviate this problem, a horizontal simplification algorithm for each curve is proposed. Computational efficiency concerns arising from the size of the datasets, such as the computation of the distance from a point to a polygon (with a huge number of vertices), and the point location problem, are addressed. To obtain an efficient implementation of the proposed method, it was necessary to use adequate data structures and computational geometry algorithms, in order to solve several subproblems, for instance: the computation of the distance from a point to a polygon (with a huge amount of vertices), the simplification of the level curves and the point location problem. Finally, we show how to visualize the interpolant corresponding to the n-th resolution level.

1. Introduction

A digital terrain model (DTM) is a representation of the Earth's surface in digital form, without including details such as vegetation, bridges, buildings, etc. It is very useful to visualize the terrain in a computer and to study its properties. DTMs are widely used in engineering and planning applications such as land claims, soil movement in construction, optimal localization of radio broadcasting stations, determination of potential flooding areas and the study of morphological parameters.

If we know the terrain height for a discrete set of points and we assume that the Earth's surface is a bivariate function, then a DTM can be seen as an approximation to this function. The height measurements can be obtained using different techniques, the most common data distributions are grid (or raster), scattered and contour lines. The selection of the DTM strongly depends on data distribution. For instance, the Triangular Irregular Network model is used when the data are scattered. The contour lines are constructed joining the set of data points with equal height in such a way that contour lines are suitable approximations of terrain isolines.

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0377-0427/$ – see front matter
doi:10.1016/j.cam.2012.08.003

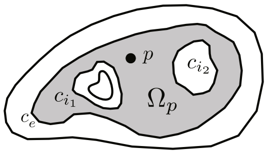

Fig. 1. Level curves, interest region Ω_p associated with a point p, exterior boundary c_e and interior boundaries c_{i_1} and c_{i_2}.

Each representation has advantages and disadvantages. Contour lines are very popular among cartographers as they provide a picture of the 3D terrain using only a planar map. Irregular triangulations are most suitable for 3D visualization because the current 3D visualization is based on triangles. Level curves may be obtained directly or computed as the isolines of a surface fitting the initial data. In both cases, the results are polygonal curves with many edges which approximate the isolines. The resolution in the vertical direction is variable and depends on the source of data points and the parameters of the approximation function. The visualization of the terrain in real time is very expensive due to the huge number of data points in a relatively small area. In consequence, it becomes very convenient to search a new representation of the terrain which approximates the initial data using significantly less points.

In this paper we propose a new method for computing a multiresolution representation of a digital terrain model using the level curves. To obtain the model we organize the level curves according to their "importance". This classification provides a mechanism for increasing or reducing the level of details depending on what is the priority: precision or speed. The core of the method is an algorithm to eliminate level curves. It allows us to obtain a sequence of approximations of the terrain, where the difference between two consecutive approximations is only one curve: the less important.

2. Previous work

Terrain model reconstruction is usually done by means of different types of spline functions. The most commonly used are: thin plate splines [1], radial functions [2,3], intermediate contour estimation [4] and linear splines [5,6]. Most of these approximation functions do not provide directly a multiresolution representation. More precisely, the interpolation methods defined in terms of these functions supply an approximation of the height at points outside the original measure set, but they do not define necessarily a mechanism to increase or decrease the level of details by adding or removing data. An exception are linear splines, which allow us to adapt mesh simplification techniques to obtain a sequence of increasingly rougher representations, [7]. However, most of the previous techniques are not prepared for the data coming from DTM, which makes them unnecessarily complex for the problem at hand.

In [8], Floater proposes a general strategy for terrain simplification. Unlike his previous work [9], the Adaptive Thinning strategy uses the height information from the data set to compute the simplified surfaces. The proposed algorithm is iterative, at each step i corresponding to the set S_i, a subset $S_{i+1} \subset S_i$ is computed which minimizes the error among all the subsets S'_{i+1} that can be obtained by removing one element from S_i.

In [10] a strategy to interpolate a disperse set using tensor product B-Splines is presented. The scheme divides the region into rectangles, which are used as support for a C^1-continuous approximation surface S_1. The rectangular regions where the error $\|h(p) - S_1(p)\|$ is higher than a threshold, are further subdivided and another function S_2 is computed to approximate the function $h(p)-S_1(p)$. The process continues until a desired error is achieved. The result is a surface $S = S_1+S_2+S_3+\cdots$, where each term increases the level of detail over one of the sub-rectangles. The main disadvantage of using tensor product splines is that they are only able to represent correctly those details lined up with the main directions. A similar approach is presented in [11,12], where triangular patches are used instead of rectangular patches, obtaining more degrees of freedom to approximate the details. These methods are more suitable for surface editing since they allow us to add details on the surface interactively.

As far as we know, none of the previous multiresolution strategies makes use of the level-curve information. Thus, the main goal of this paper is to develop a multiresolution representation of a terrain which takes advantage of the level-curve information.

3. Preliminaries

In this section we introduce the basic concepts necessary in the rest of the paper, see Fig. 1.

Definition 3.1 (*Level Curve*)**.** It is a polygonal curve c which approximates a connected component isoline of the terrain. It can be a closed or open curve, in the latter case the extreme points lie on the boundary of the region. We denote by S the set of level curves.

The Definition 3.1 is necessary since we only know the height value of the terrain at a finite set of points, therefore the isolines are approximated by polygonal curves. Moreover, in practical problems we work with rectangular maps, in consequence some isolines, restricted to the rectangular region, are open curves, see Fig. 2, left and center. Furthermore, due to errors in measurements or preprocessing of data, sometimes we have level curves that are not continuous or open level curves with extreme points that don't belong to the boundary of the map. To solve these problems, we make the level curves continuous