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Lossless and near-lossless image compression based on multiresolution analysis[☆]

Francesc Aràndiga*, Pep Mulet, Vicent Renau

Departament de Matemàtica Aplicada, Universitat de València, Spain

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ABSTRACT

There are applications in data compression, where quality control is of utmost importance. Certain features in the decoded signal must be exactly, or very accurately recovered, yet one would like to be as economical as possible with respect to storage and speed of computation. In this paper, we present a multi-scale data-compression algorithm within Harten's interpolatory framework for multiresolution that gives a specific estimate of the precise error between the original and the decoded signal, when measured in the L_∞ and in the L_p $(p = 1, 2)$ discrete norms.

The proposed algorithm does not rely on a tensor-product strategy to compress two-dimensional signals, and it provides *a priori* bounds of the Peak Absolute Error (PAE), the Root Mean Square Error (RMSE) and the Peak Signal to Noise Ratio (PSNR) of the decoded image that depend on the quantization parameters. In addition, after data-compression by applying this non-separable multi-scale transformation, the user has an the *exact value* of the PAE, RMSE and PSNR before the decoding process takes place.

We show how this technique can be used to obtain lossless and near-lossless image compression algorithms.

1. Introduction

Image compression plays a very important role in many technology driven applications, for example document and medical imaging, tele videoconferencing, etc. Image compression techniques have often been classified into two categories: lossless and lossy schemes [1–4]. Lossless techniques are typically chosen for applications where small image details are very important, these include medical and space imaging or remote sensing, to name a few. Lossy techniques, on the other hand, are used in situations where significant compression ratios are needed. This is the case, for example, in digital photography, for which, generally, loosing some image detail is not necessarily problematic.

A third class of techniques has attracted the attention of image coding researchers: near-lossless image coding. Near-lossless compression algorithms strive to provide a significant increase in compression rates while providing, at the same time, quantitative estimates on the type and amount of distortion introduced by the compression algorithm [5–8]. In this paper we shall describe how to use certain multiresolution transformations in order to design lossless and near-lossless compression techniques.

Multiscale transformations are being used as the first step of transform coding algorithms for image compression. A multiresolution-based compression scheme can be schematically described as follows: Let $\bar{f}^L$ be a sequence of data, sampled at a given resolution. Its multiresolution representation, $M\bar{f}^L = (\bar{f}^0, d^1, \ldots, d^L)$, is composed by $\bar{f}^0$, which corresponds to a

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* Corresponding author.
E-mail addresses: arandiga@uv.es (F. Aràndiga), mulet@uv.es (P. Mulet), vireto@alumni.uv.es (V. Renau).

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sampling of the data at a much coarser resolution, and a sequence of details, d^k, representing the intermediate information which is necessary to recover $\bar{f}^k$, the data sampled at level k, from $\bar{f}^{k-1}$.

The basic argument for data compression is that fact that $M\bar{f}^L$ is a much more *efficient* representation of the image data, i.e. its components can be quantized, or truncated, with little loss of information contents. $M\bar{f}^L$ is then *processed*, using a (non-reversible) quantizer $Q_{\bar{\varepsilon}}M\bar{f}^L = (\hat{f}^0, \hat{d}^1, \ldots, \hat{d}^L)$ and passed on to the encoder, which produces the final compressed set of data which is ready to be transmitted or stored. Under the basic assumption of *efficiency* of the multi-scale representation, compression is indeed achieved during the second and third steps: the quantization and the encoding of the transformed set of discrete data.

The strategy used to obtain $Q_{\bar{\varepsilon}}M\bar{f}^L$ is user-dependent and it is usually designed so that $M\bar{f}^L$ and $Q_{\bar{\varepsilon}}M\bar{f}^L$ are *close*, in the sense that after the processing $\|\bar{f}^0 - \hat{f}^0\| \le \varepsilon_0$ and $\|\hat{d}^k - d^k\| \le \varepsilon_k$, $1 \le k \le L$, $\bar{\varepsilon} = \{\varepsilon_0, \ldots, \varepsilon_L\}$, for some prescribed discrete norm, where ε_i are the *quantization* parameters. Decoding the compressed representation requires the application of M^{-1}, the inverse multiresolution transformation, to the *modified* data $Q_{\bar{\varepsilon}}M\bar{f}^L$. This step leads to $\hat{f}^L = M^{-1}Q_{\bar{\varepsilon}}M\bar{f}^L$, a set of data which is expected to be *close* to the original discrete set $\bar{f}^L$. A bound of the type

$$\|\bar{f}^L - \hat{f}^L\| \le C\varepsilon \tag{1}$$

is expected to hold, for some $C > 0$, an appropriate norm and for ϵ related to the quantization sequence $\{\epsilon_i\}$. This estimate is directly related to the *stability* of the inverse transformation [9–13].

Many multiresolution-based compression algorithms ensure stability, i.e. that (1) holds after decoding, but very often cannot provide specific stability bounds that are of practical use. Often, stability is proven for the L_2 norm, as this is the natural norm for wavelet-based compression algorithms. However, in some applications it is most important to be able to control the L_∞ norm of the decoded image. This seems to be a fundamental issue in applications involving medical and digital elevation maps (DEMs) [1,5,7]. A large error in an individual pixel, or a small area in the image, might render the decoded image useless in DEM applications like navigation and landing.

In general, it is not easy to find compression algorithms that ensure a prescribed level of quality on the decoded signal. As mentioned before, in many wavelet-based compression algorithms (1) is obtained as a consequence of certain properties of the basis functions, and explicit bounds are, very often, not given. In addition, these bounds usually provide gross over-estimations of the actual errors, and are of little practical use.

In contrast, the so-called error-control algorithms within Harten's framework for multiresolution, can be shown to provide explicit bounds on the compression error. Originally designed to ensure stability of the inverse multiresolution transformation for nonlinear prediction schemes, Harten's error-control algorithms rely on a *modified direct transform* which can *track* the errors committed in the processing step. In [14], a modified multiresolution transformation based on a tensor product extension of the one-dimensional case developed by Harten [9] is proposed and analyzed. In particular, in the interpolatory setting, it is proven in [14], that the compression error $\|\bar{f}^L - \hat{f}^L\|$ (in the L_∞, L_1 and L_2 discrete norms) can be *exactly estimated*, before the compressed signal is actually decoded, from certain quantities that are computed along with the transformation.

For data compression of two-dimensional signals, it is desirable to work with non-separable multiscale transformations, which allow the use of fully 2D prediction operators (see [15–17]). Although the cell-averages framework is more adequate for image processing [18,15,17], in some cases the point-values framework let us have applications that cannot be obtained in the cell-averages framework. For instance, within the cell-average framework it is not possible to obtain a lossless compression algorithm.

In this paper we develop a non-separable modified multiscale transformation in the point-value framework that can be seen as an extension of the original tensor product error-control algorithm developed in [14]. In [19,20] we presented the algorithm without details and a theorem giving bounds in the max-norm between the original and the reconstructed images. In [20] we also presented a technique which enable us to obtain lossless compression. Here we show *thoroughly* how to use it. We also shall provide exact estimates in the L_p norms, $p = 1, 2$. Our results allows us to provide *a priori* bounds of the Peak Absolute Error (PAE), the Root Mean Square Error (RMSE) and the Peak Signal to Noise Ratio (PSNR) in terms of the quantization parameter. In addition, the ability to estimate the exact compression error, in various norms, of our algorithm can be used to provide the exact PAE, RMSE and PSNR before the decoding takes place. Other novelty of this work is that this has been exploited to improve the compression rate, whilst keeping a prescribed PAE.

We have organized the paper as follows: In Section 2 we describe our multi-scale framework: the 2D point-value (interpolatory) framework for multiresolution. In Section 3 we present our non-separable modified direct transform and prove the theorems that give us explicit error bounds for data-processing in our non-separable two-dimensional multi-scale transformations. Some numerical experiments on the use of these multiscale transformations for lossless and near-lossless compression are performed in Section 4. Finally, some conclusions and perspectives for future work are drawn in Section 5.

2. Multiscale transformations in the point value setting

Harten's framework for multiresolution is a relatively well known subject these days. It has been described and analyzed in the literature (see e.g. [9–21] and references therein). Here, we are interested in the interpolatory framework, which we shall briefly describe for notational purposes in the following.