Journal of Computational and Applied Mathematics 242 (2013) 248–260

Contents lists available at SciVerse ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A compound Poisson risk model with proportional investment☆

Xu Chen*, Hui Ou

College of Mathematics and Computer Science, Key Laboratory of High Performance Computing and Stochastic Information Processing, Ministry of Education of China, Hunan Normal University, Changsha, Hunan 410081, PR China

ARTICLE INFO

Article history:
Received 10 July 2012
Received in revised form 21 October 2012

Keywords:
Discounted dividend payments
Gerber–Shiu discounted penalty function
Integro-differential equation
Sinc numerical methods

ABSTRACT

This paper considers the compound Poisson risk model with a threshold dividend strategy and proportional investment. The goal here is to investigate the expected discounted dividend payments and the expected penalty–reward function. Integro-differential equations with certain boundary conditions are derived. As closed-form solutions do not exist, a numerical sinc method is proposed. Finally, some examples illustrating the procedure are presented.

1. Introduction

In the past decade, investment problems for various risk models have gained much interest. An important topic is the risk models with stochastic return. The pioneer work can be traced to Segerdahl in [1]. Since then many research studies have been carried out on this topic; see e.g., [2–5]. The other important topic is the optimal control problem. In this area, the objective of the company is to find a policy, consisting of risk control and a dividend payment scheme, which maximizes the expected total discounted dividend payments until the time of bankruptcy. This is a mixed regular-singular control problem which has received renewed interest recently; e.g., [6–8].

Different from the existing research results, in this paper we consider the compound Poisson surplus model with threshold dividend strategy and proportional investment and study the expected discounted dividend payments and the expected penalty–reward function. The concept of the expected discounted penalty function introduced by Gerber and Shiu has proven to be a powerful analytical tool to study ruin quantities; see [9,10]. The expected discounted dividend payment is an important quantity which the shareholders of insurance company concern with. Although some elegant results have been obtained for the expected discounted dividend payments and the expected penalty–reward function, see [11,12], the case with investment still needs further investigation.

We begin by introducing the classical Cramér–Lundberg risk model. The surplus process of an insurance portfolio $\{U(t)\}_{t\geq 0}$ is given by

$$U(t) = u + ct - S(t) = u + ct - \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0 \tag{1}$$

where $u = U(0) \geq 0$ is the initial surplus level, and $c > 0$ is the constant premium income rate. The aggregate claims process $\{S(t)\}_{t\geq 0}$ is a compound Poisson process, comprising a homogeneous Poisson process $\{N(t)\}_{t\geq 0}$ with rate $\lambda > 0$

☆ The work is supported by NSF of China, Grant Nos 11171101, 10871064 and Doctoral Fund of edu. ministry20104306110001.
* Corresponding author. Tel.: +86 15111372026.
E-mail address: chenxu981388@yahoo.com.cn (X. Chen).

0377-0427/$ – see front matter
doi:10.1016/j.cam.2012.10.027

defined as $N(t) = \sup\{k : T_1 + T_2 + \cdots + T_k \le t\}$, and independent and identically distributed (i.i.d) claim sizes $\{Y_i\}_{i=1}^{\infty}$ that are distributed as a generic continuous random variable (r.v.) with c.d.f. F_Y and p.d.f. $f_Y(\cdot)$. The i.i.d inter-claim times $\{T_i\}_{i=1}^{\infty}$ have a common exponential distribution with the parameter λ.

Let us allow the insurer to invest its surplus in a financial market consisting of a risk-free asset (bond or bank account) and a risky asset (stock or mutual fund). Specially, the price process of the risk-free asset $\{R_t\}_{t\ge 0}$ is given by

$$dR_t = rR_t dt, \tag{2}$$

where $r > 0$ is the risk-free interest rate. The risky asset price process $\{P_t\}_{t\ge 0}$ follows a geometric Lévy process given by

$$P_t = e^{X(t)}, \tag{3}$$

$$X(t) = at + \sigma W_t + \sum_{i=1}^{M(t)} X_i, \tag{4}$$

where a and σ are positive constants that represent the expected instantaneous rate of return of the risky asset and the volatility of the risky asset price, respectively. $\{W_t, t \ge 0\}$ is a standard Brownian motion. $\{X_i, i = 1, 2, \ldots\}$ are assumed to be i.i.d random variables with c.d.f. $F_X(\cdot)$ and p.d.f. $f_X(\cdot)$. $\{M(t), t \ge 0\}$ is the Poisson process with intensity γ, and is defined as $M(t) = \sup\{k : S_1 + S_2 + \cdots + S_k \le t\}$, where $\{S_i\}_{i=1}^{\infty}$ are the i.i.d inter-jump times of $X(t)$ with a common exponential distribution. In addition, $\{Y_i, i = 1, 2, \ldots\}$, $\{X_i, i = 1, 2, \ldots\}$, $\{N(t), t \ge 0\}$, $\{M(t), t \ge 0\}$ and $\{W(t), t \ge 0\}$ are all independent. Clearly, the price process $\{P_t, t \ge 0\}$ satisfies the following stochastic differential equation,

$$\frac{dP_t}{P_t} = \left(a + \frac{1}{2}\sigma^2\right) dt + \sigma dW_t + d\sum_{i=1}^{M(t)} (e^{X_i} - 1). \tag{5}$$

In addition we use $0 < q < 1$ to denote the proportion of the surplus being invested in the risky asset. The remainder proportion, $1 - q$, is held in the risk-free asset. Then, the surplus process takes the form

$$dU(t) = qU(t-)\frac{dP_t}{P_t} + (1-q)U(t-)\frac{dR_t}{R_t} + cdt - dS_t. \tag{6}$$

In this paper, we consider the risk model (6) in which dividends are paid according to a threshold dividend strategy. Under the threshold dividend strategy, whenever the surplus is above $b > 0$, dividends are paid continuously at a constant rate $\alpha (0 < \alpha \le c)$, however when the surplus is below the level b, no dividends are paid. Incorporating the barrier strategy into (6) yields the surplus process $\{U_b(t), t \ge 0\}$,

$$\begin{aligned} dU_b(t) &= \begin{cases} qU_b(t-)\dfrac{dP_t}{P_t} + (1-q)U_b(t-)\dfrac{dR_t}{R_t} + cdt - dS_t, & U_b(t-) < b \\ qU_b(t-)\dfrac{dP_t}{P_t} + (1-q)U_b(t-)\dfrac{dR_t}{R_t} + (c-\alpha)dt - dS_t, & U_b(t-) \ge b \end{cases} \\ &= \begin{cases} q\sigma U_b(t-)dW_t + (r'U_b(t-) + c)dt + qU_b(t-)d\displaystyle\sum_{i=1}^{M(t)}(e^{X_i} - 1) - d\sum_{i=1}^{N(t)} Y_i, & U_b(t-) < b \\ q\sigma U_b(t-)dW_t + (r'U_b(t-) + c - \alpha)dt + qU_b(t-)d\displaystyle\sum_{i=1}^{M(t)}(e^{X_i} - 1) - d\sum_{i=1}^{N(t)} Y_i, & U_b(t-) \ge b \end{cases} \end{aligned} \tag{7}$$

where $r' = q(\frac{1}{2}\sigma^2 + a) + (1-q)r$, and the net profit condition is given by $c - \alpha > \lambda E[Y_1]$.

Let $D(t)$ be the cumulative amount of dividends paid out up to time t and $\delta > 0$ the force of interest. Thus

$$D_{u,b} = \int_0^{T_b} e^{-\delta t} dD(t) = \alpha \int_0^{T_b} e^{-\delta t} I(U_b(t) > b) dt, \tag{8}$$

is the present value of all dividends until T_b, where T_b denoted by $T_b = \inf\{t : U_b(t) \le 0\}$ is the time of ruin, where $I(\cdot)$ denotes the indicator function. It is obvious that $0 < D_{u,b} \le \frac{\alpha}{\delta}$. For $u \ge 0$, we use the symbol $V(u; b)$ to denote the expectation of $D_{u,b}$,

$$V(u; b) = E[D_{u,b} | U_b(0) = u].$$

The remainder of the paper is organized as follows. In Section 2, we derive the respective integro-differential equation systems for $V(u; b)$ and the expected penalty-function. In Section 3 a numerical method to approximate the solution of the integro-differential equation system via the sinc-collocation method is considered. In Section 4 we give some numerical examples to portray how the investment proportion "q" affect the expected discounted dividend payments and ruin probability.