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Numerical and dimensional investigation of two-phase countercurrent imbibition in porous media

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ABSTRACT

In this paper, we introduce a numerical solution of the problem of two-phase immiscible flow in porous media. In the first part of this work, we present the general conservation laws for multiphase flows in porous media as outlined in the literature for the sake of completion where we emphasize the difficulties associated with these equations in their primitive form and the fact that they are, generally, unclosed. The second part concerns the 1D computation for dimensional and non-dimensional cases and a theoretical analysis of the problem under consideration. A time-scale based on the characteristic velocity is used to transform the macroscopic governing equations into a non-dimensional form. The resulting dimensionless governing equations involved some important dimensionless physical parameters such as Bond number Bo, capillary number Ca and Darcy number Da. Numerical experiments on the Bond number effect is performed for two cases, gravity opposing and assisting. The theoretical analysis illustrates that common formulations of the time-scale forces the coefficient $Da^{1/2}/Ca$ to be equal to one, while formulation of dimensionless time based on a characteristic velocity allows the capillary and Darcy numbers to appear in the dimensionless governing equation which leads to a wide range of scales and physical properties of fluids and rocks. The results indicate that the buoyancy effects due to gravity force take place depending on the location of the open boundary.

1. Introduction and background

Solving problems related to transport phenomena in porous media has imposed a great challenge to scientists and researchers for a long time in trying to find the appropriate framework in which these phenomena may be solved (see, for example, [1–7]). The ingenuity of Darcy, in our opinion, may be that, by abandoning the need for detailed, comprehensive descriptions of the flow field within the pore structures, he showed that it suffices to know an integral behavior of the system for the purpose of solving engineering design problems. This was a turning-point idea that determined the direction of research in this field in the decades to come. There have been two approaches in research for further development. The first approach starts by postulating the validity of the Darcy law [1] and then using it for further, more complex formulations. As an example, transport phenomena in variably saturated porous media are usually described by what is called the Richards equation (see [8]) which is a combination of the Darcy law for unsaturated media and the mass conservation principle. In

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this equation, the medium is characterized by two functional relationships: (1) the hydraulic conductivity curve, which gives the relationship between (unsaturated) hydraulic conductivity and pressure head, and (2) the retention curve, which gives the relationship between water content and pressure head. The second approach, on the other hand, was motivated by the fact that if this relationship (i.e., Darcy's relation) were to bear any degree of usefulness, it should be possible, in principle, to derive it based on some sort of mathematical and physical reasoning. Both schools of thought, however, agreed upon the existence of certain constraints for this relationship to be utilized. These constraints have been invoked by our understanding of the requirements for proper upscaling, [9,10]. It has been found that these requirements can easily be satisfied in simple systems of statistically homogeneous porous media. In more complex systems (e.g., fractured rock systems, unsaturated porous media, etc.) more sophisticated conceptualizations needed to be devised. However, most of them relied on the attempts to adopt the continuum hypothesis in the system under investigation. As an example, some systems have been conceptualized assuming the existence of two or more overlapping continua that are interacting with each other. These include, for example, the dual porosity model, the dual permeability model, etc.

2. Macroscopic conservation laws for multiphase flows in porous media

For multiphase flows in porous media, the problem becomes even harder by the existence of deformable interface boundaries. These interface boundaries were dealt with in the past as boundaries of discontinuity with no thermodynamic quantities associated with them (e.g., [11–14]). Later theories have incorporated the effects of interface boundaries into the macroscopic behavior of the multiphase systems by assigning to them thermodynamic properties (e.g., [15,16], and others). Different approaches were used by different authors in deriving conservation laws for multiphase systems. This includes the theory of volume averaging, theory of mixtures, homogenization theories, etc. Nevertheless, all these approaches necessitate the existence of a macroscopic length scale, l, which is larger enough than the microscopic length scale characterizing the system, d, and smaller enough than the global length scale characterizing the domain, L. In other words, it is required that

$$d \ll l \ll L. \tag{1}$$

For a single phase system, the governing conservation equations are usually adopted at a global scale and then using the divergence and Reynolds transport theories, microscopic point equations are derived. Therefore, in the framework of [17], consider a global volume V bounded by a surface boundary Ω which may have a velocity, ϖ. Applying the conservation laws for an intensive, single phase property, ψ one obtains:

$$\frac{d}{dt}\int_V \rho\psi dV + \int_\Omega \mathbf{n}\cdot\left[\rho\left(\mathbf{v}-\overline{\boldsymbol{\omega}}\right)\psi - \mathbf{i}\right]d\Omega - \int_V \rho f dV = \int_V G dV, \tag{2}$$

where ρ is the fluid density, $\mathbf{v}$ is the fluid velocity, $\mathbf{i}$ is the diffusive flux across the boundary, $\mathbf{n}$ is the outwardly unit normal vector to Ω, f is an external supply and G accounts for production of ψ within the volume. The above equation may be transformed to a point equation by applying the transport theorem and the divergence theorem. By applying these two theories, one obtains:

$$\frac{d\rho\psi}{dt} + \nabla\cdot\rho\mathbf{v}\psi - \nabla\cdot\mathbf{i} - \rho f = G. \tag{3}$$

Now for a multiphase system, the continuum mixture theories may be adopted in the same sense by introducing a global balance equation and then extracting a point equation for each phase. In this approach, each mixture component is dealt with as a continuum and therefore at each point there exists a mixture of phases. The governing global equation for each phase therefore is expected to be similar to the above mentioned equations albeit with some differences. For example, each conservative quantity is expected to be encountered in relation with the volume fraction of each phase, ε^α. Moreover, the production term, G, in the above equations, while being zero in a single phase system, may not be zero in a multiphase system, [17]. In fact it accounts for the possible interaction between phases at the macroscopic level. Therefore, [17], introduced the global conservation laws of mixtures in the form:

$$\frac{d}{dt}\int_V \rho^\alpha\varepsilon^\alpha\psi^\alpha dV + \int_\Omega \mathbf{n}\cdot\left[\rho^\alpha\varepsilon^\alpha\left(\mathbf{v}^\alpha-\overline{\boldsymbol{\omega}}\right)\psi^\alpha - \varepsilon^\alpha\mathbf{i}\right]d\Omega - \int_V \rho^\alpha\varepsilon^\alpha f^\alpha dV = \int_V \varepsilon^\alpha G^\alpha dV, \tag{4}$$

from which, likewise, the point macroscale balance equation may be introduced as:

$$\frac{d\rho^\alpha\varepsilon^\alpha\psi^\alpha}{dt} + \nabla\cdot\rho^\alpha\varepsilon^\alpha\mathbf{v}^\alpha\psi^\alpha - \nabla\cdot\mathbf{i}^\alpha - \rho^\alpha\varepsilon^\alpha f^\alpha = \varepsilon^\alpha G^\alpha. \tag{5}$$

Although this approach may generally be applied to interfaces, it can prove very difficult as outlined by Hassanizadeh and Gray [17]. Alternatively, the theory of local volume averaging provides an interesting alternative. In this method, the macroscopic equations of the multiphase systems may be obtained by applying a volume average operator to the microscopic governing equations over a representative volume whose characteristic length scale is defined in accordance