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# Nonlinear modelling of the middle ear as an elastic multibody system — Applying model order reduction to acousto-structural coupled systems

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ABSTRACT

In this study, modelling of the human hearing is considered. Due to the nonlinearity of the middle ear, the sound transfer changes as the equilibrium position of the middle ear structure varies. For the description of the middle ear a nonlinear elastic multibody system is derived. The tympanic membrane and the air in the ear canal as well as in the tympanic cavity are considered as elastic bodies. They are first modelled using the finite element method. The large number of degrees of freedom makes a following reduction step of the acousto-structural finite element model inevitable. The second-order structure of the system matrices is preserved by applying reduction techniques based on Petrov–Galerkin projection. The nonlinearity of the tympanic membrane is included following the approach of parametric model order reduction by matrix interpolation assuming that the nonlinearity can be represented by the relative pressure between the ear canal and the tympanic cavity. Finally the static and dynamic behaviour of the simulation model is reviewed for different static pressure loads of the middle ear.



## 1. Introduction

The function of the pinna, ear canal and middle ear is the transfer of sound events from the free field to the inner ear. The transfer of dynamical sound pressure is a highly dynamic process which is strongly dependent on the frequency of excitation. This pure mechanical process is determined by a coupled dynamic system consisting of the air column in the outer ear canal, the tympanic membrane (TM) and the ossicles.

Beside the dynamical sound pressure varying from 20 $\mu$Pa at the hearing threshold to 10 Pa at the pain threshold, the middle ear has to deal with large quasi static variation of preload for example induced by ambient pressure variations ($\pm$4000 Pa), the application of middle ear prostheses or scar tissue. The vibrations of the ossicular chain at moderate sound pressure are much smaller than the quasi static variations and can be regarded as small perturbations about the equilibrium position of the middle ear. Due to the nonlinearity of the middle ear, the sound transfer changes as the equilibrium position varies with the preload. As the nonlinear behaviour of the natural structure cannot be described with classic linear models, the challenge is to develop models describing both large quasi static variations and small physiological motions.

In this paper an elastic multibody system (EMBS) model of the middle ear is presented. In the rigid multibody system (MBS) described in [1] the TM and the adjacent air in the ear canal and the tympanic cavity are replaced by elastic bodies. The finite element method (FEM) is used to model the geometrical and material nonlinearities of the TM. Parametric model order reduction according to [2] is applied whereas the nonlinearity is approximated by single parameter dependency. First the techniques used for modelling and model reduction are described briefly. The acousto-structural coupling between the

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TM and the adjacent air is implemented using a pressure-based Eulerian approach. Hence the system of equations becomes unsymmetric and modal truncation can be applied regarding different right and left eigenvectors. Finally, the modelling of the elastic bodies is described in detail and static and dynamic simulation results of the EMBS are reviewed.

## 2. Methods

Since sound transfer is related to forces from sound pressure excitation and motions of anatomical structures, mechanical modelling is an adequate modelling technique. For analysing the dynamic behaviour of the middle ear structure the governing differential equations are derived.

The FEM gives a detailed insight into local strength and deformation but leads to a high number of degrees of freedoms. This method is used by several research groups, e.g. [3–5], whereas mostly the middle ear is modelled as a linear system neglecting the influence of the preload of the ossicular chain. Therefore the application of these models is limited to small perturbations of the ossicular chain, impeding an adequate description of the dynamic behaviour in the case of pathologies, reconstruction or preload. Since the anatomical structures in the middle ear, e.g. the ligaments and tendons, are inhomogeneous, crude assumptions have to be made to formulate adequate parameters even for the FEM description.

The multibody system modelling technique describes the considered system based on a low number of equations which are in general of nonlinear form. The benefit of the MBS technique is a relative low number of parameters which can be estimated from specific measurements on cadaver specimens or from the overall dynamical behaviour of the living system. Especially in the case of the ligaments and tendons with their inhomogeneous material properties and a geometry which is difficult to assess, the determination of adequate model parameters is easier when using a lumped parameter model. In its typical formulation an MBS consists of rigid bodies which are interconnected with ideal joints and constraint elements. Taking the elastic deformation into account one has to deal with an elastic multibody system. Mostly the FEM is used to describe the elastic deformations usually leading to a high number of elastic coordinates. Hence model reduction techniques are used to reduce the number of coordinates.

The second-order structure of the system has to be preserved for the coupling with the MBS. The acousto-structural coupling is taken into account in the FEM model using a pressure-based approach. This approach leads to unsymmetric system matrices and, therefore, modal truncation is an oblique projection since the right and left eigenvectors are different. This cannot be considered with the standard input data (SID) format, described in [6], which is used by commercial software. With Neweul-$M^2$ [7], however, it is possible to simulate two-sided (oblique) projected elastic bodies. Since one-sided projections cannot guarantee the stability of the reduced system in the case of unsymmetric system matrices [8], here modal reduction as a two-sided projection is applied. In order to include the nonlinear behaviour of the TM, several local models are generated by linearizing the nonlinear FEM model at different working points. The nonlinear elastic body is determined by weighted interpolation of the reduced local models following the approach of parametric model order reduction.

### *2.1. Elastic multibody system*

The concept of an EMBS is described in detail, e.g. in [6,9]. Here, the floating frame of reference approach is used, where the movement of an elastic body is divided into the movement of a reference frame and small elastic deformations relative to this reference frame. The equation of motion for a single elastic body is

$$\begin{bmatrix} \boldsymbol{M}_r & \boldsymbol{M}_{re} \\ \boldsymbol{M}_{er} & \boldsymbol{M}_e \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{a} \\ \ddot{\boldsymbol{q}}_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_r \\ \boldsymbol{h}_e \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{K}_e \cdot \boldsymbol{q}_e - \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}}_e \end{bmatrix}. \tag{1}$$

The rigid multibody dynamics of the elastic body is described by the upper left submatrix $\boldsymbol{M}_r$ and the global acceleration vector of the floating frame of reference $\boldsymbol{a}$. The matrices $\boldsymbol{M}_e$, $\boldsymbol{D}_e$ and $\boldsymbol{K}_e$ are the mass, damping and stiffness matrices of the flexible structure with $\boldsymbol{q_e}$, $\dot{\boldsymbol{q}}_e$ and $\ddot{\boldsymbol{q}}_e$ denoting the small elastic deformations and their derivatives. The coupling matrices $\boldsymbol{M}_{re}$ and $\boldsymbol{M}_{er}$ take into account the inertia coupling between the reference motion and the elastic deformation of the body. The volume, surface and centrifugal as well as the gyroscopic forces are summarized in the force vectors $\boldsymbol{h}_r$ and $\boldsymbol{h}_e$.

In our model the TM and the adjacent air are modelled as elastic bodies which are derived from an FEM model. For modelling the fluid–structure coupling of the TM and the adjacent air various formulations are available. Symmetric formulations can be derived from the Lagrangian approach where the motion in the fluid region is described in terms of displacements. However, that suffers from numerical instabilities due to spurious modes that occur at the modal analysis [10]. Therefore, the pressure-based Eulerian approach is used leading to the coupled system

$$\underbrace{\begin{bmatrix} \boldsymbol{M}_s & \boldsymbol{0} \\ \rho_f \boldsymbol{C}^T & \boldsymbol{M}_f \end{bmatrix}}_{\boldsymbol{M}_{fs} \in \mathbb{R}^{n \times n}} \cdot \underbrace{\begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{bmatrix}}_{\ddot{\boldsymbol{q}}_e \in \mathbb{R}^n} + \underbrace{\begin{bmatrix} \boldsymbol{D}_s & \boldsymbol{D}_{up} \\ \boldsymbol{D}_{pu} & \boldsymbol{D}_f \end{bmatrix}}_{\boldsymbol{D}_{fs} \in \mathbb{R}^{n \times n}} \cdot \underbrace{\begin{bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{p}} \end{bmatrix}}_{\dot{\boldsymbol{q}}_e \in \mathbb{R}^n} + \underbrace{\begin{bmatrix} \boldsymbol{K}_s & -\boldsymbol{C} \\ \boldsymbol{0} & \boldsymbol{K}_f \end{bmatrix}}_{\boldsymbol{K}_{fs} \in \mathbb{R}^{n \times n}} \cdot \underbrace{\begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix}}_{\boldsymbol{q}_e \in \mathbb{R}^n} = \begin{bmatrix} \boldsymbol{f}_{ex}(t) \\ \boldsymbol{p}_{ex}(t) \end{bmatrix} \tag{2}$$

with the mass matrix $\boldsymbol{M}_{fs}$, the damping matrix $\boldsymbol{D}_{fs}$ and the stiffness matrices $\boldsymbol{K}_{fs}$. The matrices are unsymmetric, whereas the indices $s$ and $f$ denote the solid and fluid partition, respectively. The matrix $\boldsymbol{C}$ couples the displacements $\boldsymbol{u}$ with the pressure variations $\boldsymbol{p}$ and $\rho_f$ is the density of the fluid. The elastic coordinates $\boldsymbol{q_e}$ contain both structural and acoustic quantities, with