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Asymptotic stability of balanced methods for stochastic jump-diffusion differential equations[☆]

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ABSTRACT

Positive results are proved here about the ability of balanced methods to reproduce the asymptotic stability of the stochastic differential equation with jumps. Balanced methods including strong balanced methods and weak balanced methods, which possess implicitness in the diffusion term, have the potential to overcome some of the numerical instabilities that are often experienced when using the explicit methods. The paper shows that the asymptotic stability for stochastic jump-diffusion differential equations is inherited by the two kinds of balanced methods with sufficiently small stepsizes. Some numerical experiments included in the paper illustrate the theoretical results.

1. Introduction

The stochastic model has come to play an important role in many branches of science and engineering. Such models have been used with great success in a variety of application areas, including epidemiology, mechanics, economics and finance. In certain areas, such as finance, the uncertainty in the dynamics is, in fact, the essential phenomenon that needs to be modeled. Event-driven dynamics has become more and more important in most fields of application and leads to the stochastic differential equations (SDEs) with jumps. Several empirical studies, including Bates [1] and Pan [2], demonstrated the existence of jumps in stock markets and bond markets. Therefore, models that incorporate jumps have become increasingly popular in finance; see, for instance, Kou [3], Schönbucher [4] and Chiarella and Nikitopoulos-Sklibosios [5]. Beyond finance there are many areas of application, including electrical engineering and biotechnology, which use jump-diffusion models (see, e.g. [6,7]).

Unfortunately, SDEs with jumps rarely have explicit solutions. Thus, appropriate numerical methods are needed to apply in practice and to study their properties. The numerical analysis of SDEs with jumps is well studied (see, e.g. [8–12]) and of the stochastic differential delay equations (SDDEs) with jumps is discussed in [13,14].

However, it is already known that the majority of the numerical methods for SDEs with jumps or SDDEs with jumps are explicit or semi-implicit methods. Semi-implicit methods are well adapted for stiff systems with small stochastic noise intensity or additive noise. But in those cases in which the stochastic part plays an essential role in the dynamics, e.g., as

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it is with large multiplicative noise, the application of fully implicit methods also involving implicit stochastic terms is unavoidable. One of the most important fully implicit numerical methods is the balanced implicit method, which was first proposed by Platen et al. [15]. In 2006, Alcock and Burrage [16] obtained the region of the asymptotic stability and the mean-square stability of balanced methods for SDEs. Furthermore, Tan et al. [17] investigated the convergence and stability of balanced methods for SDDEs. Lately, Wang and Liu [18] discussed the convergence and stability of the split-step backward balanced Milstein methods for SDEs. Moreover, Kahl and Schurz [19] introduced the balanced Milstein methods with strong order 1.0.

Consider the following scalar linear jump-diffusion stochastic differential equation

$$dx(t) = ax(t^-)dt + bx(t^-)dW(t) + cx(t^-)dN(t), \quad t > 0 \tag{1.1}$$

with $x(0^-) = x_0$. Here $x(t^-)$ denotes $\lim_{s\to t^-} x(s)$, $x_0 \neq 0$ with probability one, the coefficients $a, b, c \in \mathbb{R}$ and $c \neq 0$. Here, $W(t)$ is a scalar Brownian motion independent of $N(t)$ which is a scalar Poisson process with intensity $\lambda(\lambda > 0)$, both defined on an appropriate complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$, with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e. it is increasing and right-continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets).

It is known that (1.1) has the analytic solution [20]

$$x(t) = x_0(1+c)^{N(t)} \exp\left[\left(a - \frac{1}{2}b^2\right)t + bW(t)\right]. \tag{1.2}$$

Regarding stability analysis, Higham and Kloeden [10] discussed the mean-square stability of the strong theta method and the weak theta method for the system (1.1).

Lately, Chalmers and Higham [21] have given the necessary and sufficient condition for the stochastically asymptotic stability in the large (hereafter, asymptotic stability) of the system (1.1). Furthermore, they have studied the asymptotic stability of the theta-method approximation.

There is also some work concerned with the weak approximation schemes for SDEs without jumps; see, for example, Kloeden and Platen [22], Saito and Mitsui [23], Burrage et al. [24] and Cao and Liu [25].

However, to the best of our knowledge, there are no stability results of balanced methods for SDEs with jumps. The aim of this paper is to investigate the asymptotic stability of balanced methods. In Section 2, we discuss the asymptotic stability of strong balanced methods. Section 3 deals with the asymptotic stability of weak balanced methods. In addition, some numerical results are reported in Section 4.

2. Asymptotic stability of strong balanced methods

Given a stepsize $h > 0$, a version of strong balanced methods for (1.1) is given by

$$\begin{cases} Y_{n+1} = Y_n + aY_nh + bY_n\Delta W_n + cY_n\Delta N_n + C(Y_n)(Y_n - Y_{n+1}), & n \geq 0, \\ Y_0 = Y(0), \end{cases} \tag{2.1}$$

where Y_n is an approximation to $x(t_n)$ with $t_n = nh$, $Y(0) = x(0)$, $\Delta W_n = W(t_{n+1}) - W(t_n)$, $\Delta N_n = N(t_{n+1}) - N(t_n)$ and ΔW_n is independent of ΔN_n. Here $C(Y_n)$ is given by

$$C(Y_n) = C_n = C_0(Y_n)h + C_1(Y_n)|\Delta W_n| = C_{0n}h + C_{1n}|\Delta W_n|, \tag{2.2}$$

where the $C_{0n} = C_0(Y_n)$, $C_{1n} = C_1(Y_n)$ are called control functions. In order to obtain our main results in this paper, we assume that C_{0n}, C_{1n} in Eq. (2.2) are constants, that is, $C_{0n} = C_0$, $C_{1n} = C_1$.

Assumption 1. For any real numbers $\alpha_0 \in [0, \overline{\alpha}]$, $\alpha_1 \geq 0$, where $\overline{\alpha} \geq h$ for all step sizes h considered, the constants C_0, C_1 satisfy $1 + \alpha_0 C_0 + \alpha_1 C_1 \neq 0$.

It is known that the balanced methods (2.1) give strong convergence rate of at least 1/2; see, for example, [26].

We say the balanced methods (2.1) are asymptotically stable for a particular choice of a, b, c, λ and h if $\lim_{n\to\infty} |Y_n| = 0$, with probability one, for any Y_0.

Lemma 2.1 gives the necessary and sufficient condition for the asymptotic stability of the system (1.1).

Lemma 2.1 ([21]). *The exact solution $x(t)$ of the system (1.1) is asymptotically stable if and only if*

$$a - \frac{1}{2}b^2 + \lambda \ln|1+c| < 0, \tag{2.3}$$

where $\ln|1+c| = -\infty$, as $c = -1$. Thus when $c = -1$, (2.3) means that the system (1.1) is asymptotically stable for any $a, b \in \mathbb{R}$.

The following lemma is important in the proof of Theorem 2.1. In the similar way as Theorem 3.2 in [27] we can obtain the following lemma.