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Strong predictor–corrector Euler–Maruyama methods for stochastic differential equations with Markovian switching

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ABSTRACT

In this paper numerical methods for solving stochastic differential equations with Markovian switching (SDEwMSs) are developed by pathwise approximation. The proposed family of strong predictor–corrector Euler–Maruyama methods is designed to overcome the propagation of errors during the simulation of an approximate path. This paper not only shows the strong convergence of the numerical solution to the exact solution but also reveals the order of the error under some conditions on the coefficient functions. A natural analogue of the p-stability criterion is studied. Numerical examples are given to illustrate the computational efficiency of the new predictor–corrector Euler–Maruyama approximation.

1. Introduction

Stochastic differential equations with Markovian switching (SDEwMSs) arise in mathematical models of hybrid systems that possess frequent unpredictable structural changes. One of the distinct features of such systems is that the underlying dynamics are subject to change with respect to certain configurations. Such models have been used with great success in a variety of application areas, including flexible manufacturing systems, electric power networks, risk theory, financial engineering and insurance modeling; we refer the readers to Cheng et al. [1], Ghosh et al. [2], Jobert and Rogers [3], Mao and Yuan [4], Rolski et al. [5], Smith [6], Wu et al. [7], Yang and Yin [8], Zhao et al. [9] and references therein.

Generally, although the fundamental theories such as the existence and uniqueness of the solution as well as stability of SDEwMSs have been well studied, most of SDEwMSs cannot be solved analytically. Thus, appropriate numerical approximation methods such as the Euler (or Euler–Maruyama) method are needed to apply SDEwMSs in practice or to study their properties.

Yuan and Mao [10] first considered the numerical solutions of the following stochastic differential equation with Markovian switching

$$dy(t) = f(y(t), r(t))dt + g(y(t), r(t))dW(t), \quad t \geq 0 \tag{1.1}$$

with initial conditions $y(0) = y_0 \in \mathbb{R}^d$ and $r(0) = r_0 \in S = \{1, 2, \ldots, N\}$, f and g are sufficiently smooth so that Eq. (1.1) has a unique solution. Here $y(t)$ is referred to the state while $r(t)$ is regarded as the mode. The system will switch from one mode to another in a random way, and the switching between the modes is governed by a Markov chain. They proved the mean-square convergence of the Euler–Maruyama (EM) approximation for this hybrid stochastic systems, and the order of error was also estimated. Yin et al. [11] extended (1.1) to a family of more general jump-diffusions with Markovian switching,

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and proved the numerical solutions based on the finite-difference procedure converge weakly to the desired limit by means of a martingale problem formulation.

During recent years, there also exist extensive studies which prove the convergence of the Euler–Maruyama method applied to some stochastic differential equation with some additional features, like including predictor–corrector or linear-implicit methods and including some sort of delay, jumps, Markovian switching or combinations thereof; see for example, Bruti-Liberati and Platen [12,13], Hou et al. [14], Li and Hou [15], Mao and Yuan [4], Rathinasamy and Balachandran [16], Schurz [17–19] among others. The corresponding proof is basically the same each time, the only novelty coming from changing it a bit to deal with the additional feature.

It is well known that the Euler–Maruyama method and most other explicit schemes for solving stochastic differential equations (SDEs) work unreliably and sometimes generate large errors; see for instance Klauder and Petersen [20], Petersen [21], Milstein et al. [22]. Implicit and predictor–corrector schemes are designed to achieve improved numerical stability and turn out to be better suited to simulation task. Generally, implicit schemes usually cost significant computational time and are sometimes not reliably accomplished; however, this phenomenon can be avoided when using some appropriate discrete time schemes, including predictor–corrector methods. In Kloeden and Platen [23], predictor–corrector methods have been proposed as weak discrete time approximations for solving SDEs, which can be used in Monte Carlo simulation. For the strong discrete time approximation of solutions of SDEs, a family of predictor–corrector Euler methods has been developed by Bruti-Liberati and Platen [13]. However, there are no strong predictor–corrector methods available for SDEwMSs yet.

In this paper, we develop a new family of strong predictor–corrector Euler–Maruyama (PCEM) methods for SDEwMS (1.1), which are shown to converge with strong order 0.5, and demonstrate their performance by considering some examples.

The rest of the paper is arranged as follows. In Section 2 we introduce some necessary notations and define a family of strong predictor–corrector Euler–Maruyama approximate solutions to SDEwMSs. In Section 3 we show that the PCEM solutions converge to the exact solution in L^2 under the global Lipschitz condition and reveal that the order of convergence is 0.5. In Section 4 we extend the PCEM convergence results to multi-dimensional case under certain conditions. In Section 5 the numerical stability of SDEwMSs will be introduced and discussed. In Section 6 some numerical examples are given and compared for simulated paths with different degrees of implicitness to illustrate the computational efficiency of the predictor–corrector Euler–Maruyama approximation. Finally, some concluding remarks and future works are provided in Section 7.

2. Preliminary and algorithm

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Suppose that there is a finite set $S = \{1, 2, \dots, N\}$, representing the possible regimes of the environment. We work with a finite-time horizon $[0, T]$ for some $T > 0$.

Consider the dynamic system given by (1.1) with initial values $y(0) = y_0 \in \mathbb{R}^d$ and $r(0) = i_0 \in S$, where $f(\cdot,\cdot) : \mathbb{R}^d \times S \to \mathbb{R}^d$, $g(\cdot,\cdot) : \mathbb{R}^d \times S \to \mathbb{R}^{d\times m}$, $W = \{W(t) = (W^1(t), \dots, W^m(t))^T, t \geq 0\}$ is an m-dimensional $\mathcal{F}_t$-adapted Wiener process, and $r = \{r(t), t \geq 0\}$ is a continuous-time Markov chain taking value in a finite state space S with the generator $Q = (q_{ij})_{N\times N}$ given by

$$P\{r(t+\delta) = j | r(t) = i\} = \begin{cases} q_{ij}\delta + o(\delta), & \text{if } i \neq j, \\ 1 + q_{ii}\delta + o(\delta), & \text{if } i = j, \end{cases}$$

provided $\delta \downarrow 0$, and

$$-q_{ii} = \sum_{i\neq j} q_{ij} < +\infty.$$

We assume that W and r are independent. Throughout this paper, we denote by $|\cdot|$ the Euclidean norm for vectors and $\|\cdot\|$ the trace norm for matrices.

2.1. Existence and uniqueness

Under certain conditions we can establish the existence of a pathwise unique solution of (1.1). Here we make the following global Lipschitz (GL) and linear growth (LG) assumptions.

($\mathcal{H}1$) GL. There exists a constant $L_1 > 0$, for all $(x, i), (y, i) \in \mathbb{R}^d \times S$, such that

$$|f(x,i) - f(y,i)|^2 + \| g(x,i) - g(y,i) \|^2 \leq L_1 |x-y|^2.$$

($\mathcal{H}2$) LG. There exists a constant $L_2 > 0$, for all $(x, i) \in \mathbb{R}^d \times S$, such that

$$|f(x,i)|^2 \vee \| g(x,i) \|^2 \leq L_2(1 + |x|^2).$$

Remarks 2.1. It is easy to show that if $f(\cdot,\cdot)$, $g(\cdot,\cdot)$ satisfy the GL condition, then they also satisfy the LG condition, but, for the convenience of our later statements, we explicitly require it here.