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Tridiagonal implicit method to evaluate European and American options under infinite activity Lévy models

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ABSTRACT

We propose an efficient implicit method to evaluate European and American options when the underlying asset follows an infinite activity Lévy model. Since the Lévy measure of the infinite activity model has the singularity at the origin, we approximate infinitely many small jumps by samples of a diffusion. The proposed methods to solve partial integro–differential equations for European options and linear complementarity problems for American options via an operator splitting method involve solving linear systems with tridiagonal matrices and so can significantly reduce the computations associated with the discrete integral operators. The numerical experiments verify that the proposed method has the second-order convergence rate under an infinite activity Lévy model.

1. Introduction

Recently, to overcome the drawbacks of the Black–Scholes model, the infinite activity Lévy models have drawn much attention from many researchers as alternative models that can explain some stylized facts on the return process of a stock in the financial market such as negative skewness, heavy tails, volatility smiles, and have been widely applied to modeling various financial assets including equity models, fixed income models, and structural models for credit derivatives [1,2]. Among the parsimonious infinite activity Lévy models, the variance gamma model introduced by Madan and Seneta [3] and the normal inverse Gaussian model suggested by Barndorff-Nielsen [4] are based on Brownian subordination. The generalized hyperbolic model proposed by Eberlein et al. [5] is the normal variance–mean mixture where the mixing distribution is the generalized inverse Gaussian distribution. The CGMY model introduced by Carr et al. [6] is constructed by specifying the Lévy measure which generalizes that of the variance gamma model.

In this paper, we are interested in numerical methods to evaluate European and American options under infinite activity Lévy models. The prices of American options can be obtained by solving linear complementarity problems (LCPs) while the prices of European options are given by solving partial integro–differential equations (PIDEs). In order to evaluate European options under the infinite activity models, Carr and Madan [7] proposed a method with the fast Fourier transform (FFT) when the characteristic function of the log return is known analytically. Cont and Voltchkova [8] presented an explicit–implicit method with the first-order convergence rate to obtain the numerical approximation of viscosity solutions for European

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and barrier options under the Merton and variance gamma models. Hirsa and Madan [9] developed a numerical method for American options under the variance gamma model. They treated the integral term, which has singularity at the origin, by dividing the domain of the integration into six components. Almendral and Oosterlee [10] suggested the second-order backward differentiation formula (BDF2) with an extrapolation to evaluate American options under the variance gamma model.

In the case of the CGMY model in [6], there are various methods [11–13] for pricing American options. Almendral and Oosterlee [11] transformed the partial integro–differential operator into that in terms of Volterra operators with a weakly singular kernel and applied a collocation method for Volterra equations to the integral operator. This numerical method shows the possibility to achieve second-order convergence rate with a finite variation. Wang et al. [13] proposed an implicit timestepping method in which they avoided dense linear systems by using either a fixed point iteration or a BiCGSTAB method. The implicit method has the second-order accuracy for the finite variation and better than the first-order accuracy for the infinite variation. Rambeerich et al. [12] suggested an exponential time integration (ETI) method to evaluate American options. The ETI method requires the computation of a matrix exponential. The traditional implicit numerical methods however require the computation of the inverse of a dense matrix that comes from a discrete integral operator with known and unknown values mixed at every time level, which leads to very computationally intensive iterations at every time level.

The objective of this paper is to propose an efficient implicit finite difference method to evaluate European and American options under the infinite activity model with both the finite variation and the infinite variation. We apply the implicit method with three time levels in [14,15] to solve the PIDE for European options and the LCP for American options. In order to treat the inequality constraints in the LCP, we use the operator splitting method in [16] among a variety of methods. We focus on the construction avoiding iteration at each time step and then our implicit method is based on three time levels. Since we use linear systems whose coefficient matrices are tridiagonal matrices, the computational cost can be improved. We present numerical results that show the second-order convergence rate and the plots of the early exercise boundary and the delta.

The rest of this paper is organized as follows. In Section 2 we introduce the exponential infinite activity model. In Section 3 we describe the implicit method with three time levels to evaluate European and American options under the infinite activity models. We apply the implicit method combined with the operator splitting method for the American option. In Section 4 we carry out some numerical experiments for pricing European and American options under the CGMY model. The paper ends with conclusions in Section 5.

2. The exponential infinite activity Lévy model

Let us consider a stock price process of an underlying asset S_t that follows an exponential Lévy model, that is, $S_t = S_0 \exp((r-d)t + X_t)$ on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$, where S_0 is the stock price at $t = 0$, r is the risk-free interest rate, d is the continuous dividend yield, and X_t is a Lévy process with a Lévy triplet (σ^2, γ, ν). The Lévy measure ν satisfies the following condition

$$\int_{|x|<1} |x|^2 \nu(dx) < \infty \quad \text{and} \quad \int_{|x|\geq 1} \nu(dx) < \infty.$$

In a risk-neutral world, there exists an equivalent martingale measure $\mathbb{Q}$ under which $(S_t e^{-rt})_{t\geq 0}$ is a martingale. Then it gives the following conditions

$$\int_{|x|>1} e^x \nu(dx) < \infty \quad \text{and} \quad \gamma = -\frac{1}{2}\sigma^2 - \int_{\mathbb{R}} (e^x - 1 - x 1_{\{|x|\leq 1\}}) \nu(dx),$$

where 1_A is the indicator function with respect to a set A.

An infinite activity Lévy process is described by an infinite number of jumps, that is, $\nu(\mathbb{R}) = \infty$. As an example of the infinite activity processes, the CGMY process is defined by specifying the Lévy measure ν as follows

$$\nu(dx) = C\left(\frac{e^{-G|x|}}{|x|^{1+Y}} 1_{x<0} + \frac{e^{-Mx}}{x^{1+Y}} 1_{x>0}\right) dx, \tag{1}$$

where $C > 0, G > 0, M > 0$, and $Y < 2$. The condition $Y < 2$ ensures that the Lévy measure integrates x^2 in the neighborhood of the origin. The CGMY process X_t is introduced by Carr et al. [6]. When the parameter Y is zero, it is well-known for the variance gamma (VG) process proposed by Madan and Seneta [3]. The VG process is obtained by the time changed Brownian motion with drift where time is given by a gamma process. For other types of infinite activity Lévy processes, please refer to [17].

3. Option pricing under the infinite activity model

Since the infinite activity Lévy model has an infinite number of small jumps (e.g. for $0 \leq Y < 2$ in the case of CGMY model), its Lévy measure ν is singular at the origin. In this section, we discuss the option pricing problems under the infinite activity model following Cont and Voltchkova [8].