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A backward parabolic equation with a time-dependent coefficient: Regularization and error estimates

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ABSTRACT

We consider the problem of determining the temperature $u(x, t)$, for $(x, t) \in [0, \pi]\times[0, T)$ in the parabolic equation with a time-dependent coefficient. This problem is severely ill-posed, i.e., the solution (if it exists) does not depend continuously on the given data. In this paper, we use a modified method for regularizing the problem and derive an optimal stability estimation. A numerical experiment is presented for illustrating the estimate.

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1. Introduction

Let T be a positive number, we consider the backward problem for the nonhomogeneous linear parabolic equation

$$u_t(x, t) - a(t)u_{xx}(x, t) = f(x, t), \quad (x, t) \in [0, \pi] \times (0, T] \tag{1}$$

$$u(0, t) = u(\pi, t) = 0, \quad t \in [0, T] \tag{2}$$

$$u(x, T) = g(x), \quad x \in [0, \pi] \tag{3}$$

where $a(t)$ is a function such that there exists $p, q > 0$

$$0 < p \le a(t) \le q. \tag{4}$$

Many physical and engineering problems in areas require the solution of the backward problem for the parabolic equation with a time-dependent coefficient. In general, the backward problem for the parabolic equation is ill-posed in the sense that the solution (if it exists) does not depend continuously on the given data. It means that a small perturbation on the data can affect the exact solution largely. Hence, it is difficult to calculate the regularized solution closing the exact solution and a regularization is necessary. In fact, the linear case has been studied in the past four decades by many scientists all over the

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world. Moreover, there were so many papers relating to the backward problem for the parabolic equation (see, e.g., [1–7]). In [8], the authors introduced a method which was called the quasi-reversibility method (QR method). They regularized the problem by using a "corrector term" in order to add it into the main equation. In a particular case, they investigated the problem

$$u_t + Ku - K^*Ku = 0,$$
$$u(x, T) = g(x).$$

We see that the above problem is useful if we can construct the adjoint operator K^*. In fact, the other approximated problem was more practical than the problem given in [9,10]

$$u_t + Ku - Ku_t = 0,$$
$$u(x, T) = g(x).$$

On the other hand, in 1983, Showalter presented the quasi-boundary value method (QBV method). By using the QBV method, they regularized the problem by adding the "corrector term" into the final condition. Applying this method, Dense and Bessila [2] used the final condition as follows

$$u(x, T) - \epsilon u_x(x, 0) = g(x).$$

As stated above, there are many works on the backward problem for the parabolic equation with a constant coefficient, the paper related to the time-dependent coefficient is very scarce. Recently, in [11], the authors consider the backward problem for the heat equation (with a constant coefficient) and obtain the error estimates between the regularized solution and the exact solution as follows

$$\|u^\epsilon(.,t) - u(.,t)\| \le C\epsilon^{\frac{t}{T}}, \quad \text{for } t > 0$$
$$\|u^\epsilon(.,0) - u(.,0)\| \le \sqrt[4]{8}C\sqrt[4]{T}\,(\ln(1/\epsilon))^{-\frac{1}{4}}, \quad \text{for } t = 0.$$

We can easily see that the above estimate tends to zero slowly when t is in a neighborhood of zero. That is the one disadvantage of this method (using in [11]). However, in [12], by requiring some acceptable assumptions of f and the exact solution u, the authors also improved the method (using in [12]) in order to obtain the better error estimate than [11]

$$\|u^\epsilon(.,t) - u(.,t)\| \le T_1(1+\sqrt{M})\exp\left\{\frac{3L^2TT_1^2(T-t)}{2}\right\}\epsilon^{t/T}\ln\left(\frac{T}{\epsilon}\right)^{\frac{t}{T}-1}.$$

Hence, in this paper a modified method is given for regularizing the backward problem with the time-dependent coefficient and obtain the error estimate that tends to zero more quickly than the logarithmic order.

In this paper, we also approximate (1)–(3) by using the regularization problem

$$u^\epsilon(g)(x,t) = \sum_{m=1}^{\infty}\left[\frac{\exp\{-m^2F(t)\}}{\beta+\exp\{-m^2F(T)\}}g_m - \int_t^T \frac{\exp\{m^2(F(s)-F(t)-F(T))\}}{\beta+\exp\{-m^2F(T)\}}f_m(s)ds\right]\sin(mx) \tag{5}$$

where

$$g_m = \frac{2}{\pi}\int_0^\pi g(x)\sin(mx)dx,$$
$$f_m(s) = \frac{2}{\pi}\int_0^\pi f(x,s)\sin(mx)dx,$$
$$F(t) = \int_0^t a(s)ds,$$

and $\beta = \beta(\epsilon)$ (denoting by β) are chosen later. The rest of this paper is divided into two sections. In Section 2, the regularization results and the proof of main results are presented. A numerical experiment is shown in Section 3 to illustrate the main results.

2. Regularization and error estimates

For clarity, we denote that $\|.\|$ is the norm in $L^2[0, \pi]$.

2.1. Main results

In this section, we shall give the regularized solution of (1)–(3) and estimate the error between the regularized solution and the exact solution. Hence, we need to find out the exact solution of (1)–(3). In fact, the exact solution of (1)–(3) satisfies

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