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A note on the application of the generalized finite difference method to seismic wave propagation in 2D

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ABSTRACT

This paper shows the application of generalized finite difference method (GFDM) to the problem of seismic wave propagation. We investigated stability and star dispersion in 2D.

We obtained independent stability conditions and star dispersion of the phase velocity for the P and S waves. Also, P and S waves group velocity dispersion have been obtained.

1. Introduction

The generalized finite difference method (GFDM) is evolved from classical finite difference method (FDM). GFDM can be applied over general or irregular clouds of points. The basic idea is to use moving least squares (MLS) approximation to obtain explicit difference formulae which can be included in partial differential equation to establish, together with an explicit method, a recursive relationship. The authors have made many contributions to the development of this method [1–7].

In this paper, this meshless method is applied to seismic wave propagation. Stability conditions and grid dispersion relations in 2D are derived.

2. Explicit generalized difference schemes for the seismic wave propagation problem for a perfectly elastic, homogeneous and isotropic medium

2.1. Equation of motion

The equation of motion and Hooke's law for a perfectly elastic, homogeneous, isotropic medium in 2D are

$$\begin{cases} \dfrac{\partial^2 U(x,y,t)}{\partial t^2} = \alpha^2 \dfrac{\partial^2 U(x,y,t)}{\partial x^2} + \beta^2 \dfrac{\partial^2 U(x,y,t)}{\partial y^2} + (\alpha^2 - \beta^2) \dfrac{\partial^2 V(x,y,t)}{\partial x \partial y} \\ \dfrac{\partial^2 V(x,y,t)}{\partial t^2} = \beta^2 \dfrac{\partial^2 V(x,y,t)}{\partial x^2} + \alpha^2 \dfrac{\partial^2 V(x,y,t)}{\partial y^2} + (\alpha^2 - \beta^2) \dfrac{\partial^2 U(x,y,t)}{\partial x \partial y} \end{cases} \tag{1}$$

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with the initial conditions

$$
\begin{aligned}
&U(x,y,0)=f_1(x,y); \qquad V(x,y,0)=f_2(x,y)\\
&\frac{\partial U(x,y,0)}{\partial t}=f_3(x,y); \qquad \frac{\partial V(x,y,0)}{\partial t}=f_4(x,y)
\end{aligned}
\tag{2}
$$

and the boundary condition

$$
\begin{cases}
a_1U(x_0,y_0,t)+b_1\dfrac{\partial U(x_0,y_0,t)}{\partial n}=g_1(t)\\
a_2V(x_0,y_0,t)+b_2\dfrac{\partial V(x_0,y_0,t)}{\partial n}=g_2(t)
\end{cases}
\quad \text{en } \Gamma
\tag{3}
$$

where $f_1(x,y), f_2(x,y), f_3(x,y), f_4(x,y), g_1(t)$ y $g_2(t)$ are known functions,

$$
\alpha=\sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad \beta=\sqrt{\frac{\mu}{\rho}}
$$

ρ is the density, λ and μ are Lamé elastic coefficients and Γ is the boundary of Ω.

2.2. Explicit generalized difference schemes

The aim is to obtain explicit linear expressions for the approximation of partial derivatives in the points of the domain. First of all, an irregular grid or cloud of points is generated in the domain $\Omega \cup \Gamma$. On defining the central node with a set of nodes surrounding that node, the star then refers to a group of established nodes in relation to a central node. Every node in the domain has an associated star assigned to it.

Following [1,3,5–7], the explicit difference formulae for the spatial derivatives are obtained,

$$
\begin{cases}
\dfrac{\partial^2 U(x_0,y_0,n\Delta t)}{\partial t^2}=\dfrac{u_0^{n+1}-2u_0^n+u_0^{n-1}}{(\Delta t)^2}\\
\dfrac{\partial^2 V(x_0,y_0,n\Delta t)}{\partial t^2}=\dfrac{v_0^{n+1}-2v_0^n+v_0^{n-1}}{(\Delta t)^2}
\end{cases}
\tag{4}
$$

$$
\begin{aligned}
&\frac{\partial^2 U(x_0,y_0,n\Delta t)}{\partial x^2}=-m_0u_0^n+\sum_{j=1}^{N}m_ju_j^n; \qquad \frac{\partial^2 V(x_0,y_0,n\Delta t)}{\partial x^2}=-m_0v_0^n+\sum_{j=1}^{N}m_jv_j^n\\
&\frac{\partial^2 U(x_0,y_0,n\Delta t)}{\partial y^2}=-\eta_0u_0^n+\sum_{j=1}^{N}\eta_ju_j^n; \qquad \frac{\partial^2 V(x_0,y_0,n\Delta t)}{\partial y^2}=-\eta_0v_0^n+\sum_{j=1}^{N}\eta_jv_j^n\\
&\frac{\partial^2 U(x_0,y_0,n\Delta t)}{\partial x\partial y}=-\zeta_0u_0^n+\sum_{j=1}^{N}\zeta_ju_j^n; \qquad \frac{\partial^2 V(x_0,y_0,n\Delta t)}{\partial x\partial y}=-\zeta_0v_0^n+\sum_{j=1}^{N}\zeta_jv_j^n
\end{aligned}
\tag{5}
$$

where N is the number of nodes in the star whose central node has the coordinates (x_0, y_0) (in this work $N = 8$ and the are selected by using the four quadrants criteria [1,6]).

m_0, η_0, ζ_0 are the coefficients that multiply the approximate values of the functions U and V at the central node for the time $n\Delta t$ (u_0^n and v_0^n respectively) in the generalized finite difference explicit expressions for the space derivatives.

m_j, η_j, ζ_j are the coefficients that multiply the approximate values of the functions U and V at the rest of the star nodes for the time $n\Delta t$ (u_j^n and v_j^n respectively) in the generalized finite difference explicit expressions for the space derivatives.

The replacement in Eq. (1) of the explicit expressions obtained for the partial derivatives leads to

$$
\begin{cases}
u_0^{n+1}=2u_0^n-u_0^{n-1}+(\Delta t)^2\Bigg[\alpha^2\left(-m_0u_0^n+\displaystyle\sum_1^N m_ju_j^n\right)+\beta^2\left(-\eta_0u_0^n+\displaystyle\sum_1^N \eta_ju_j^n\right)\\
\qquad +(\alpha^2-\beta^2)\left(-\zeta_0v_0^n+\displaystyle\sum_1^N \zeta_jv_j^n\right)\Bigg]\\
v_0^{n+1}=2v_0^n-v_0^{n-1}+(\Delta t)^2\Bigg[\beta^2\left(-m_0v_0^n+\displaystyle\sum_1^N m_jv_j^n\right)+\alpha^2\left(-\eta_0v_0^n+\displaystyle\sum_1^N \eta_jv_j^n\right)\\
\qquad +(\alpha^2-\beta^2)\left(-\zeta_0u_0^n+\displaystyle\sum_1^N \zeta_ju_j^n\right)\Bigg].
\end{cases}
\tag{6}
$$