Journal of Computational and Applied Mathematics 236 (2012) 2706–2716

Contents lists available at SciVerse ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

The hitting time for a Cox risk process

Rong Wu [a], Wei Wang [b,*]

[a] School of Mathematical Sciences and LPMC, Nankai University, Tianjin 300071, PR China
[b] College of Mathematical Science, Tianjin Normal University, Tianjin, 300387, PR China

ARTICLE INFO

Article history:
Received 22 November 2009
Received in revised form 2 January 2012

MSC:
60J75
91B30

Keywords:
Cox process
Classical risk process
The hitting time
The last exit time

ABSTRACT

This paper investigates the hitting time of a Cox risk process. The relationship between the hitting time of the Cox risk process and the classical risk process is established and an explicit expression of the Laplace–Stieltjes transform of the hitting time is derived by the probability method. Similarly, we derive the explicit expression of the Laplace–Stieltjes transform of the last exit time. Further, we study the situation when the intensity process is an *n-state* Markov process.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

Collective risk theory, as the most important theoretical part in actuarial mathematics, mainly deals with stochastic models arising from the insurance business. We refer the readers to [1,2] for a systematical study of collective risk theory. In this paper, we consider an important quantity called the first hitting time. The study of the first hitting time for a risk process is important both in itself and for its applications. For example, [3,4] are two recent reviews concerning game theory where the topic of risk and exit times, especially with regards to the Markov process is also frequently addressed.

Let $(\Omega, \mathscr{F}, P)$ be a complete probability space containing all objects defined in the following. Consider a kind of Cox risk process $\{U(t), t \geq 0\}$ introduced in [1]:

$$U(t) = u + \mu(1+\rho)\int_0^t \lambda(s)ds - \sum_{k=1}^{N(t)} Z_k, \quad t \geq 0, \tag{1.1}$$

where $u \geq 0$ is the insurer's initial capital, $\rho > 0, Z = \{Z_k, k \geq 1\}$ is a sequence of *i.i.d.* random variables representing the claim amount, having common distribution function $F(x)$ with density function $f(x)$ and mean value μ. $N = \{N(t), t \geq 0\}$ is a Cox process with intensity process $\lambda = \{\lambda(t), t \geq 0\}$.

We call the counting process $\{N(t)\}$ a Cox process if $N(t) = \widetilde{N}(\int_0^t \lambda(s)ds)$, where $\widetilde{N} = \{\widetilde{N}(t), t \geq 0\}$ is a standard Poisson process with intensity one, $\lambda(t) \geq 0$, P-a.s., $\widetilde{N}$ and λ are independent.

In this paper we assume that $\widetilde{N}$, λ and $\{Z_k, k \geq 1\}$ are independent of each other. For simplicity, denote $c = \mu(1+\rho), A(t) = \int_0^t \lambda(s)ds$, then (1.1) becomes

$$U(t) = u + cA(t) - \sum_{k=1}^{N(t)} Z_k, \quad t \geq 0. \tag{1.2}$$

* Corresponding author.
E-mail addresses: yulinww@yahoo.com.cn, nksxww@yahoo.com.cn (W. Wang).

0377-0427/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2012.01.010

Note that $A(t)$ is a non-decreasing process with P-a.s. $A(0) = 0, A(t) < \infty$ for each $t < \infty$ and $A(\infty) = \infty$. $cA(t)$ can be interpreted as the premium income in the time interval $[0, t]$.

When the intensity process $\{\lambda(t)\}$ equals a constant Λ, the Cox risk model (1.2) becomes the classical risk model

$$U(t) = u + c\Lambda t - \sum_{k=1}^{N(t)} Z_k, \quad t \geq 0. \tag{1.3}$$

In this case, the Cox process N becomes a standard Poisson process with intensity Λ.

In practice it turns out that a Poisson claim arrival process is not realistic. Thus the natural generalization of the classical risk model is a model where N becomes a Cox process. Much work has been done under this framework, for example [5–11]. These research mainly concentrate their attention on the ruin probability and related problems. In this paper, we consider other interesting problems, the hitting time and the last exit time for the Cox risk process. For arbitrary level $b \geq u$, define the hitting time of b point to be

$$T_b = \begin{cases} \inf\{t > 0, U(t) = b\}, \\ \infty, \quad \text{if the above set is empty.} \end{cases}$$

For $\delta > 0$, let $L(u; b) = E[e^{-\delta T_b} | U(0) = u]$ be the Laplace–Stieltjes (L–S) transform of T_b.

Define the last exit time (also called quitting time) of zero level to be

$$\sigma_0 = \begin{cases} \sup\{t > 0, U(t) = 0\}, \\ 0, \quad \text{if the above set is empty.} \end{cases}$$

For $\delta > 0$, let $\Psi_0(u) = E[e^{-\delta\sigma_0} | U(0) = u]$ be the L–S transform of σ_0.

It is known that both T_b and σ_0 are important quantities in risk theory. For example, the hitting time T_b plays an important role in dividend problems. The last exit time of zero level σ_0 can be used to evaluate the ultimately leaving time of ending the negative surplus. Gerber and Shiu [12] gave the explicit expression of the L–S transform of the hitting time T_b for the classical risk process in terms of one root of Lundberg's fundamental equation. Chiu and Yin [13] studied the L–S transform of σ_0 under the spectrally negative Lévy process framework.

Now we consider the case $\Lambda = 1$, a classical risk model described by $\{\widetilde{U}(t), t \geq 0\}$:

$$\widetilde{U}(t) = u + ct - \sum_{k=1}^{\widetilde{N}(t)} Z_k, \quad t \geq 0 \tag{1.4}$$

where $c = \mu(1 + \rho)$ and $\rho > 0$ is the relative safety loading, then $P(\lim_{t\to\infty} \widetilde{U}(t) = \infty) = 1$.

It can be seen from (1.2) and (1.3) that

$$U(t) = \widetilde{U}(A(t)), \quad t \geq 0.$$

Thus we have $P(\lim_{t\to\infty} U(t) = \infty) = 1$.

Denote $\widetilde{T}_b$ the hitting time of b point and $\widetilde{\sigma}_0$ the last exit time of zero level for the classical risk process $\{\widetilde{U}(t), t \geq 0\}$. Then we have

$$A(T_b) = \widetilde{T}_b, \qquad A(\sigma_0) = \widetilde{\sigma}_0. \tag{1.5}$$

Define the inverse A^{-1} of A by

$$A^{-1}(t) = \sup\{s : A(s) \leq t\}.$$

The time scale defined by A^{-1} is generally called the operational time scale. Then

$$T_b = A^{-1}(\widetilde{T}_b), \qquad \sigma_0 = A^{-1}(\widetilde{\sigma}_0). \tag{1.6}$$

Our paper is organized as follows. In Section 2, we first give some results for the classical risk model. Since the intensity measure $A(t)$ plays an important role in the relationship between the Cox risk model and the classical risk model, we derive the Laplace transform of $A^{-1}(t)$ in this section. Then we can derive an explicit expression for $L(u; b)$ by the probability method. Using the same method as in Section 3, we derive an explicit expression for $\Psi_0(u)$. In Section 4, we study the case that the intensity process is an *n-state* Markov process. A detailed discussion is given for the *two-state* Markov process.

2. An explicit expression for $L(u; b)$

In this section, we derive an explicit expression of $L(u; b)$.

2.1. Some results for the classical risk model (1.3)

Wu et al. [14] studied $\widetilde{T}_0$ when $u \geq 0$. Now we give some preliminary results for $\widetilde{T}_b$.

Download English Version:

https://daneshyari.com/en/article/4639679

Download Persian Version:

https://daneshyari.com/article/4639679

Daneshyari.com