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Hybrid linear and nonlinear complexity pursuit for blind source separation☆

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ABSTRACT

Blind source separation (BSS) is an increasingly popular data analysis technique with many applications. Several methods for BSS using the statistical properties of original sources have been proposed; for a famous case, non-Gaussianity, this leads to independent component analysis (ICA). In this paper, we propose a hybrid BSS method based on linear and nonlinear complexity pursuit, which combines three statistical properties of source signals: non-Gaussianity, linear predictability and nonlinear predictability. A gradient learning algorithm is presented by minimizing a loss function. Simulations verify the efficient implementation of the proposed method.

1. Introduction

Blind source separation (BSS) [1,2] is an emerging data analysis technique used in many practical applications such as speech and image processing, biomedical signal processing, wireless telecommunication systems, economic data analysis, data mining, etc. The main objective of BSS is to recover unknown original source signals from their mixtures without knowing the mixing channels, using some statistical properties of the original sources. Suppose that n unknown sources are mixed simultaneously in a linear mixing channel modeled as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t), \tag{1}$$

where $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))^T$ denotes the n-dimensional observation vector, $\mathbf{A}$ is the $n \times n$ unknown nonsingular constant mixing matrix, and $\mathbf{s}(t) = (s_1(t), \ldots, s_n(t))^T$ is the n-dimensional vector of unknown zero-mean and unit-variance original sources. The task of BSS is to recover the sources $\mathbf{s}(t) = (s_1(t), \ldots, s_n(t))^T$ from the mixtures $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))^T$.

The BSS problem has been studied by researchers in applied mathematics, neural networks and statistical signal processing. Several methods for BSS using the statistical properties of original sources have been proposed; areas encompassed have included non-Gaussianity (or equivalently, independent component analysis, ICA) [1–10], and time–structure information, such as linear predictability or smoothness [1,11], linear autocorrelation [12–14], coding complexity [15–19], temporal predictability [20], nonstationarity [21–23], energy predictability [24], nonlinear innovation [25], nonlinear autocorrelation [26–28], etc.

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In this paper, we present a hybrid technique for BSS based on linear and nonlinear complexity pursuit, which combines three statistical properties of source signals: non-Gaussianity, linear predictability and nonlinear predictability. First, we propose a contrast function for BSS based on the hybrid technique, and we perform the optimization using a gradient descent algorithm (Section 2). Then, we show how the contrast function is connected to other BSS contrast functions (Section 3). Simulation results show that the model separates sources in cases where existing methods are not able to do so (Section 4), and finally we conclude the paper (Section 5).

2. The proposed algorithm

Assume that the measured sensor signals $\mathbf{x}$ have already been followed by an $n \times n$ whitening matrix $\mathbf{V}$ such that the components of $\tilde{\mathbf{x}}(t) = \mathbf{V}\mathbf{x}(t)$ are of unit variance and uncorrelated. Furthermore, assume that we want to estimate a source signal; for this purpose we design a single processing unit described as

$$\begin{aligned} \tilde{y}_i(t) &= \mathbf{w}_i^T \tilde{\mathbf{x}}(t) \\ \tilde{y}_i(t-\tau) &= \mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau), \end{aligned} \tag{2}$$

where $\mathbf{w}_i = (w_{i1}, \ldots, w_{in})^T$ is the weight vector which corresponds to the estimate of one row of $(\mathbf{VA})^{-1}$, $\tilde{y}_i(t)$ is the output signal which corresponds to the estimate of the source signal s_i, and τ is some lag constant, often equal to 1.

We present the following constrained minimization problem based on the contrast function with the non-Gaussianity, the linear predictability and the nonlinear predictability of the desired source:

$$\begin{aligned} \min_{\|\mathbf{w}_i\|=1} \Psi(\mathbf{w}_i) &= \lambda E\{G(\tilde{y}_i(t) - \alpha \tilde{y}_i(t-\tau))\} + (1-\lambda) E\{G(f(\tilde{y}_i(t)) - \beta f(\tilde{y}_i(t-\tau)))\} \\ &= \lambda E\{G(\mathbf{w}_i^T \tilde{\mathbf{x}}(t) - \alpha \mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau))\} + (1-\lambda) E\{G(f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t)) - \beta f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)))\}. \end{aligned} \tag{3}$$

In this optimization problem, $\tilde{y}_i(t) - \alpha \tilde{y}_i(t-\tau)$ and $f(\tilde{y}_i(t)) - \beta f(\tilde{y}_i(t-\tau))$ define the linear and nonlinear innovation functions of the desired source. f defines the nonlinear predictability of the desired source, and examples of choices are $f(u) = u^2$ or $f(u) = \frac{1}{\gamma}\log(\cosh(\gamma u))$ (where $\gamma \geq 1$ is a constant). α and β define autoregressive coefficients. G is a given (usually convex) loss function; generally, we can choose $G(u) = u^2$, $G(u) = |u|$, or $G(u) = \frac{1}{\gamma}\log(\cosh(\gamma u))$ (where $\gamma \geq 1$ is a constant). In this paper, we adopt the loss function $G(u) = \log(\cosh(u))$, due to its better analysis properties and robustness against outliers, such as those found in ICA [1,2,11]. λ defines the coefficient of trade-off between linear and nonlinear predictability, which measures the degrees of linear and nonlinear predictability of the desired source and includes only linear or quadratic predictability as special cases ($\lambda = 0$ for nonlinear predictability and $\lambda = 1$ for linear predictability).

In fact, assuming that $\lambda = 1$, the contrast function (3) reduces to

$$\min_{\|\mathbf{w}_i\|=1} \Psi(\mathbf{w}_i) = E\{G(\mathbf{w}_i^T \tilde{\mathbf{x}}(t) - \alpha \mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau))\}, \tag{4}$$

which is the complexity pursuit contrast function for BSS presented in the paper [15,16] when the autoregressive model has just one predicting term, which combines non-Gaussianity and linear predictability for BSS. Thus, in fact, we adopt a hybrid technique for BSS based on linear and nonlinear complexity pursuit.

To perform the optimization in (3), we can use a simple gradient descent. The gradients of $\Psi(\mathbf{w}_i)$ with respect to $\mathbf{w}_i$, α, and β are obtained as

$$\begin{aligned} \frac{\partial \Psi(\mathbf{w}_i)}{\partial \mathbf{w}_i} &= \lambda E\{(\tilde{\mathbf{x}}(t) - \alpha \tilde{\mathbf{x}}(t-\tau)) g(\mathbf{w}_i^T \tilde{\mathbf{x}}(t) - \alpha \mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau))\} \\ &\quad + (1-\lambda) E\{(f'(\mathbf{w}_i^T \tilde{\mathbf{x}}(t)) \tilde{\mathbf{x}}(t) - \beta f'(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)) \tilde{\mathbf{x}}(t-\tau)) g(f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t)) - \beta f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)))\}, \end{aligned} \tag{5}$$

$$\frac{\partial \Psi(\mathbf{w}_i)}{\partial \alpha} = -\lambda E\{(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)) g(\mathbf{w}_i^T \tilde{\mathbf{x}}(t) - \alpha \mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau))\}, \tag{6}$$

$$\frac{\partial \Psi(\mathbf{w}_i)}{\partial \beta} = -(1-\lambda) E\{f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)) g(f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t)) - \beta f(\mathbf{w}_i^T \tilde{\mathbf{x}}(t-\tau)))\}, \tag{7}$$

where the function g is the derivative of G (when $G(u) = \log(\cosh(u))$, $g(u) = \tanh(u)$) and the function f' is the derivative of f. Thus, a hybrid technique based on linear and nonlinear complexity pursuit for BSS (hybrid complexity BSS: HCBSS) is obtained as follows:

Algorithm outline: HCBSS (estimating one source)

(1) Center the data to make the mean zero and whiten the data to give $\tilde{\mathbf{x}}(t)$. Choose initial values for $\mathbf{w}_i$, α and β, and suitable learning rates $\mu_{\mathbf{w}_i}$, μ_α and μ_β.