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An adaptive extrapolation discontinuous Galerkin method for the valuation of Asian options

Michael D. Marcozzi *

Department of Mathematical Sciences, University of Nevada Las Vegas, 89154-4020, United States

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ABSTRACT

We consider the approximation of the optimal stopping problem associated with ultradiffusion processes in the context of mathematical finance and the valuation of Asian options. In particular, the value function is characterized as the solution of an ultraparabolic variational inequality. Employing the penalty method and a regularization of the state space, we develop higher-order adaptive approximation schemes which utilize the extrapolation discontinuous Galerkin method in temporal space. Numerical examples are provided in order to demonstrate the approach.

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1. Introduction

In this paper, we introduce a class of temporally adaptive techniques based on a hierarchy of higher-order methods for computing the value function of optimal stopping associated with ultradiffusion processes. Adaptive methods in time are essential to an overall assessment of approximation accuracy and in particular to the application of spatial error control. To our knowledge, no such methods currently exist relative to ultradiffusion valuations or their associated nonlinear ultraparabolic operators. The key feature of our approach is that we need only to solve a linear problem at each time step in the marching scheme in order to approximate the value function. As a prototype application, we consider an example from mathematical finance in which the value of an option depends, through the payoff, on the history of the underlying security, a so-called Asian option (cf. [1] for a more general application involving optimal stochastic control of ultradiffusion process).

An ultradiffusion is a process which is isomorphic to a parameterized diffusion along a characteristic temporal trajectory; they are motivated by the realization that in many systems exogenous sources of uncertainty enter only certain components of the dynamics (cf. [2–4]). The value function of the optimal stopping problem relative to ultradiffusion process is characterized as the solution to an ultraparabolic variational inequality. Historically, an interest in ultradiffusion processes and ultraparabolic operators arose relative to the works of Kolmogorov [5,6] and Uhlenbeck and Ornstein [7] in connection with Brownian motion in phase space and Chandrasekhar [8] with respect to the theory of boundary layers. Unlike parabolic operators, however, neither the strong maximum principle nor interior *a priori* estimates, for example, hold for ultraparabolic operators (cf. [9–15]).

* Tel.: +1 702 838 8134; fax: +1 702 895 4343.
E-mail address: michael.marcozzi@unlv.edu.

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Asian options that do not exercise early have been studied in [16,17], among others. The formal development of the early exercise case by variational methods appears to be due to Ingersoll [18], subject to the restriction that the payoff admits a similarity transformation. In terms of the numerical valuation of Asian options, and without being exhaustive, we note Barraquand and Pudet's lattice approach [19], Dewynne and Wilmott's "hyperbolic boundary condition" [20], Zvan et al. viscosity methodology [21], and Bermúdez et al. duality approach [22]. Marcozzi [23] utilizes regularization and projection methods to extend the analysis of Akrivis et al. [24] for ultraparabolic equations to the ultraparabolic variational inequality associated with Asian options.

In the following, we develop the theoretical basis for the ultraparabolic variational inequality characterization of the value function of ultradiffusion optimal stopping, its approximation by a semi-linear equation though the application of the penalty method and regularization of the state space, and introduce adaptive higher-order extrapolation discontinuous Galerkin methods for the associated ultraparabolic operator. To this end, we apply a semi-implicit extrapolation method in time to the semi-discrete approximation. Extrapolation methods are known to be competitive with, and simultaneously more flexible, than Gear-type routines (cf. [25–28]). Being everywhere defined, discontinuous Galerkin methods are a natural complement to extrapolation in the context of a temporally adaptive scheme; they were first introduced in [29] for solving the neutron transport equation and the first mathematical analysis was provided in [30]. A recent survey of the discontinuous Galerkin method may be found in [31]. For convenience and familiarity, we discretize the semi-autonomous problem utilizing linear Lagrange finite elements (cf. [32]). Relative to prior work, we note that adaptive extrapolation discontinuous Galerkin methods were introduced in [33] relative to the canonical linear ultraparabolic initial boundary value problem.

The outline of this paper is as follows. In Section 2, we characterize the value function as the unique solution to an ultraparabolic variational inequality, introduce compactly supported approximation domains as well as appropriate artificial boundary and initial conditions, and consider estimates relative to the quality of the approximation. In Section 3, we discretize the canonical problem, provide asymptotic convergence estimates, and describe a simple method for step size error control. We consider the valuation of an American-style call option on the arithmetic average in Section 4 in order to demonstrate the applicability of the method. Concluding remarks are presented in Section 5.

2. Option valuation in ultradiffusion models

In Section 2.1, we describe the characterization of the value function of optimal stopping associated with a Markov ultradiffusion process as the unique solution to an ultraparabolic variational inequality. In Section 2.2, we approximate the variational inequality by a semi-linear equation through the penalization technique. We represent the penalization problem as the limit of regularized problems posed on a sequence of bounded exhausting domains in Section 2.3. The regularized problem is put into canonical form in Section 2.4. In the context of mathematical finance, the valuation of an American-style Asian option is an example of an optimal stopping problem associated with an ultradiffusion process.

2.1. Characterization of the value function

Let $(\Omega, \mathcal{F}, \{\mathcal{F}(t)\}_{t\geq 0}, \mathbb{P})$ denote a complete filtered probability space with filtration $\{\mathcal{F}(t)\}_{t\geq 0}$ satisfying the usual hypothesis that every $\mathcal{F}(t)$ contains all of the $\mathbb{P}$-null sets of $\mathcal{F}$ and is right continuous, and let $\{B(t),\ t\geq 0\}$ be a $\mathcal{F}(t)$-adapted Brownian motion on $\mathbb{R}$ (cf. [34]). Without loss of generality, we model the underlying economic uncertainty in the so-called risk-neutral economy as a $\mathcal{F}(t)$-adapted Markov ultradiffusion process $\{(\varsigma(t), x(t)),\ t\geq 0\}$ in $\mathbb{R}^2$ with cádlág (right continuous with left-hand limits) sample paths solving the stochastic differential equation

$$d\varsigma(s) = \exp[x(s)]ds, \tag{2.1a}$$

$$dx(s) = (r-\sigma^2/2)ds + \sigma dB(s), \tag{2.1b}$$

for all $s > t$, such that $x(t) = x$, $\varsigma(t) = \varsigma$, $x = \ln(S)$, and the path history of the asset price S is represented by the variable $\varsigma(s) = \int_t^s S\, d\tau$. We suppose volatility $\sigma > 0$ and risk-free rate of return $r > 0$, in which case there exists a unique strong solution of (2.1).

Along with the process $x(t)$, reward (payoff) $\Psi(t, \varsigma, x) \geq 0$, and decision variable (stopping time) ϑ, we consider the expected discounted performance index

$$J_x^{t,\varsigma}(\vartheta) = \mathbb{E}_{\mathbb{P}}\left[\exp\left[r\cdot(t-\vartheta)\right]\cdot\Psi(\vartheta, \varsigma(\vartheta), x(\vartheta))|\mathcal{F}(t)\right], \tag{2.2a}$$

and the value function of optimal stopping

$$U(t, \varsigma, x) = \sup_{\vartheta\in\mathcal{T}_{[t,T]}} J_x^{t,\varsigma}(\vartheta), \tag{2.2b}$$

where the supremum is taken over the set of all stopping times in $[t, T]$, denoted by $\mathcal{T}_{[t,T]}$. In particular, the reward $\Psi(\vartheta, \varsigma, x)$ is received if $x(t)$ is stopped at time ϑ while in state $x(\vartheta)$ with path history $\varsigma(\vartheta)$. In the context of option pricing, the valuation (2.2) represents the present value or price of an American-style Asian option that can be exercised at any time between contract inception at time $t = 0$ and contract expiry at $T > 0$; upon exercise at time $\vartheta \in [0, T]$, the contract

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