Journal of Computational and Applied Mathematics 236 (2012) 3073–3083

Contents lists available at SciVerse ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A discrete time inventory system with postponed demands

B. Sivakumar *, R. Jayaraman, G. Arivarignan

Department of Applied Mathematics and Statistics, School of Mathematics, Madurai Kamaraj University, Madurai 625021, India

ARTICLE INFO

Article history:
Received 24 January 2010
Received in revised form 1 November 2011

MSC:
90B05
60J27

Keywords:
(s, S) policy
Discrete time inventory
Discrete Markovian arrival process
Discrete phase-type distribution
Postponed demands

ABSTRACT

In this article, we consider a discrete-time inventory model in which demands arrive according to a discrete Markovian arrival process. The inventory is replenished according to an (s, S) policy and the lead time is assumed to follow a discrete phase-type distribution. The demands that occur during stock-out periods either enter a pool which has a finite capacity $N(<\infty)$ or leave the system with a predefined probability. Any demand that arrives when the pool is full and the inventory level is zero, is assumed to be lost. The demands in the pool are selected one by one, if the on-hand inventory level is above $s+1$, and the interval time between any two successive selections is assumed to have discrete phase-type distribution. The joint probability distribution of the number of customers in the pool and the inventory level is obtained in the steady state case. The measures of system performance in the steady state are derived and the total expected cost rate is also calculated. The results are illustrated numerically.

1. Introduction

Continuous review inventory system with postponed demands has received considerable attention in the last few decades. Berman et al. [1] introduced the concept of postponement of demand in the inventory system. They assumed that both demand and service rates are deterministic. Krishnamoorthy and Islam [2] considered an inventory system with Poisson demand, exponential lead time and assumed that the pooled customers are selected according to an exponentially distributed time lag. Sivakumar and Arivarignan [3] considered an inventory model in which the demands occur according to a Markovian arrival process, lead time is distributed as phase-type, life time for the items in the stock has exponential distribution and the pooled customers are selected exponentially. Paul Manuel et al. [4] dealt with an inventory system in which the positive and negative demands occur according to independent Markovian arrival processes and the lead time of the reorder, the life time of the items, the inter-selection time of customers from the pool and the reneging time points of the customers in the pool have independent exponential distributions. Sivakumar and Arivarignan [5] considered an inventory system with independent Markovian arrival processes for both positive and negative demands, exponential distribution for the lead time, exponential life times for each item in the stock and an infinite pool size.

In all the above models, the authors assumed that all the system events are monitored continuously and that the time axis is continuous. Though many inventory systems are conveniently characterized by fixed length intervals during which events occur and decisions are made, only few articles in the literature dealt with discrete time inventory models. But, there is a growing research interest in discrete time queues (see [6–9]), mainly motivated by their applications in the Computer and Communication systems where the time axis is often slotted.

The first paper on discrete time inventory models was in [10]. They analyzed a Markovian inventory model for perishable commodities, in which the arrivals of items into the system as well as the demands for these items were assumed to be

* Corresponding author. Tel.: +91 4522673926.
E-mail address: sivabkumar@yahoo.com (B. Sivakumar).

0377-0427/$ – see front matter
doi:10.1016/j.cam.2012.01.023

Notation

$\mathbb{N}_0$	$= \{0, 1, \ldots, \}$.
$[A]_{ij}$	: element/submatrix at ith row, jth column of the matrix A.
$\mathbf{e}$	: a column vector of appropriate dimension containing all ones.
$\mathbf{0}$	: a zero matrix of appropriate dimension.
I	: identity matrix of appropriate dimension.
$[A \otimes B]_{ij}$	$= [A]_{ij}B$
$A \oplus B$	$= A \otimes I_B + I_A \otimes B$, where I_A has dimension as that of A.

discrete random variables having a common support $0, 1, \ldots$. The items are assumed to have a life span of N item units. Some characteristics of this model were derived for the case of two and three age categories.

Lian and Liu [11] developed a discrete time inventory model with geometric inter-demand times and constant life time. They assumed that the demands arrive in batches and that the batch-size was random. They also assumed that the lead time was zero and full backlogging of demands. They used matrix-analytic methods to construct a discrete time Markov chain for the inventory level and they obtained a closed-form average cost function.

Abboud [12] analyzed a discrete time Markov model for production inventory systems with machine breakdowns. He assumed that the demand and production rates were constant and the production rate was greater than the demand rate. The failure time and the repair time were independently distributed as geometric and the demands that occur during stock-out were backordered.

Recently, Lian et al. [13] discussed a discrete time model for common life time inventory systems. They assumed that the demand arrives in batches according to a discrete phase-type renewal process and the lifetime of an item had a discrete PH-distribution. They assumed that the supply of the order was instantaneous and unmet demands are backordered.

In this paper, we model an inventory system with postponed demand under a discrete time setup. We assume that the demands arrive according to a discrete Markovian arrival process and that the lead time of the order, the inter-selection time of a customer from the pool have independent discrete phase-type distributions.

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model for the problem under consideration. The steady-state analysis of the model is presented in Section 3 and some important system performance measures are derived in Section 4. In Section 5, we calculate the total expected cost rate and in the final section, we provide numerical illustrations of the results.

2. Mathematical model

Consider a discrete-time inventory system in which all system related activities occur at discrete time points only. We describe the discrete time system as defined in [14]. The system is monitored at time epochs sequentially numbered $0, 1, \ldots,$ and all events which occur between epoch t and $t+1$ are assumed to occur at epoch $t+1$. The duration of the replenishment of stock is assumed to take at least one unit time. Hence the time at which an order for replenishment is placed and that at which it is received cannot be equal.

In this work, we use the concepts of discrete phase-type distribution and discrete Markovian Arrival process and hence we give an introduction and notation on these concepts. The MAP is a class of the Markov counting process introduced in [15] as a generalization of the Poisson process which is well suited for matrix analytic and numerical investigations. In conjunction with the research reported in [16,17] a convenient notation which is better suited for a general discussion than the one which was originally used was suggested. A highly accessible discussion of the MAP, with many examples, may be found in [17]. A partly expository paper discussing how the MAP can be used qualitatively to model point processes with certain "bursty" features is given in [18]. The following is a brief informal description of the DMAP, which should be adequate for the purpose of this paper. Let D be an irreducible stochastic matrix of order n and let D_0 and D_1 be two sub stochastic matrices whose sum is D such that the matrix $I - D_0$ is nonsingular. The element $[D_0]_{ij}$ represents a transition from phase i to phase j which is associated with the non occurrence of an event (such as arrival) and the element $[D_1]_{ij}$ represents a transition from phase i to phase j which is associated with an occurrence of the above event. Let $\pi D = \pi$ with $\pi\mathbf{e} = 1$. Then the rate of occurrence of the event is $\lambda = \pi D_1\mathbf{e}$, where π is the stationary probability distribution of the tpm D. The sequence of time points of these transitions form a stochastic process which is known as a discrete Markovian arrival process (DMAP) with parameters n, D_0 and D_1. We represent such a DMAP by $(D_0, D_1)_n$.

The discrete phase-type distribution was introduced in the mid 1970's, see [19]. However, more researchers have been focusing on the studies of the continuous phase-type distributions. Detailed discussions of continuous phase-type distributions can be found in [20,21]. Brief overviews of either discrete or continuous phase-type distributions and their properties can be found in [22–26] and the references therein.

We briefly describe the phase-type distribution. Consider a Markov chain with m transient states and one absorbing state, say 0. It has an associated transition probability matrix