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The $\boldsymbol{\beta}$-Meixner model

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ABSTRACT

We propose to approximate the Meixner model by a member of the $\boldsymbol{\beta}$-family introduced by Kuznetsov (2010) in [2]. The advantage of the approximation is the *semi-explicit* formulae for the running extrema under the $\boldsymbol{\beta}$-family processes which enables us to produce more efficient algorithms for pricing path dependent options through the Wiener–Hopf factors. We will explore the performance of the approximation both in an equity framework and in the credit risk setting, where we use the approximation to calibrate a surface of credit default swaps. The paper follows the approach of the study made by Schoutens and Damme (2010) in [1], where the aim was to approximate the variance gamma. We will contextualize the results by Schoutens and Damme (2010) in [1] and the ones here with respect to the approach taken by Jeannin and Pistorius (2010) in [15]. An asymptotic expression for the rate of convergence of the approximation is derived.

1. Introduction

Schoutens and Damme [1] explore the numerical performance of the $\boldsymbol{\beta}$-family introduced by Kuznetsov in [2], both in the equity and in the credit risk field, as an approximation to the variance gamma (VG) process. The VG process is a very popular model in financial mathematics that has now been around for more than 20 years. Their conclusion is that, thanks to the *semi-explicit* formulae for the running extrema under the $\boldsymbol{\beta}$-family, they are able to produce faster and more accurate results for pricing credit default swaps (CDSs). In fact, the formulae for the running extrema are derived from explicit expressions of the Wiener–Hopf factorization. Under the VG process, the CDSs are priced using a partial differential integral equation (PDIE) approach described by Cariboni and Schoutens in [3]. The prices under both processes are equivalent and hence the methodology serves as an alternative approximate algorithm.

The aim of the present paper is to reproduce the same sort of results with respect to the Meixner process. This is also a widespread model in the financial literature. In this case, the CDS spreads under Meixner model will be computed by an inverse Fourier method. More precisely, the one described by Fang et al. in [4] and based on the cosine series expansion of the density of a Lévy process, which is called COS method (see [5,6]). Recall that apart from Monte Carlo simulation, the most general methodologies for pricing path dependent options under Lévy models are PDIEs and Fourier methods. Together with the paper of Schoutens and Damme [1], the present work shows that there is a potential use of Wiener–Hopf theory to price path dependent options as an alternative for classical approaches.

The Wiener–Hopf factorization for Lévy processes has lately been receiving an increasing attention for numerical purposes since the papers of Kuznetsov [2,7] and Kuznetsov et al. [8], which describe a wide range of Lévy processes for which the Wiener–Hopf factorization is known. Some other studies have been devoted to study the numerical tractability of the Wiener–Hopf factorization to price path dependent options, see for instance the work of Kudryavtsev and

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Levendorskiĭ [9,10]. Recall that the Wiener–Hopf factors give a description of the distributions of the extrema under an independent exponential time change. It is worth remarking that explicit expressions of the Wiener–Hopf factorization were a rare result except for particular cases such as one sided Lévy processes in Rogers [11], double sided exponential processes in Kou and Wang [12] or the cases treated in Boyarchenko and Levendorskiĭ [13] or Lewis and Mordecki [14]. In the present work we will show that the asymptotic approximation in Schoutens and Damme [1] and the one described here are particular cases of the more general technique of approximating generalized hyper-exponential Lévy processes by hyper-exponential jump-diffusion models, which was used for pricing digital options with barriers in Jeannin and Pistorius [15]. We will give an asymptotic rate of convergence for the simulation of the infinite divisible distributions derived from the Wiener–Hopf factors.

The purpose of this paper is therefore twofold. From one side the results here and the ones reported in Schoutens and Damme [1] compare the Wiener–Hopf methodology with respect to the PDIE and the Fourier methods to price options depending on the extrema of the process. On the other hand, although the Wiener–Hopf approach is just valid for a particular family of processes, we will contextualize the methodology with respect to the papers of Jeannin and Pistorius [15], Kuznetsov [2,7] and Kuznetsov et al. [8], which describe a rich family of Lévy processes.

The paper is organized as follows. In Section 2 we present the Meixner model and the $\boldsymbol{\beta}$-family, we also construct the $\boldsymbol{\beta}$-M process. Section 3 will relate the present work to the general setting of Jeannin and Pistorius [15] and Kuznetsov et al. [8]. We also give the rate of convergence of the approximation. Section 4 will derive the expressions to price vanilla options and CDS showing the numeric results. We will calibrate the Meixner and the $\boldsymbol{\beta}$-M process to a surface of vanilla options using the Carr and Madan formula (see [16]). After that, we will calibrate both models to a surface of CDS spreads. The spreads are computed under the Meixner model with the COS method, and under the $\boldsymbol{\beta}$-M process with the Wiener–Hopf factorization. Finally, we conclude the paper with some remarks.

2. The $\boldsymbol{\beta}$-family and the Meixner process

Let $X = \{X_t\}_{t\geq 0}$ be a Lévy process and recall that the law of every Lévy process is characterized by the triplet (μ, σ, ν), where $\mu \in \mathbb{R}$, $\sigma \geq 0$ is the Brownian component and ν is a measure, concentrated in $\mathbb{R} \setminus \{0\}$ and such that $\int_{\mathbb{R}}(1 \wedge x^2)\nu(dx) < \infty$. More precisely, the process is described by its Lévy exponent, $\Psi_{X_1}(z)$, as

$$\varphi_{X_t}(z) = \mathbb{E}[e^{izX_t}] = e^{-t\Psi_{X_1}(z)} \quad \forall z \in \mathbb{C}.$$

The Lévy–Khintchine representation gives the relation between the Lévy exponent and the triplet (μ, σ, ν):

$$\Psi_{X_1}(z) = -i\mu z + \frac{\sigma^2}{2}z^2 - \int_{-\infty}^{\infty}(e^{izx} - 1 - izh(x))\nu(dx), \tag{1}$$

where h is the cut-off function. In the following we can consider $h(x) \equiv x$ for the Lévy measures we are interested in.

The Meixner process is a pure jump process often used in the financial literature, we refer to Schoutens [17] and the references therein for a variety of examples where this model has been used. The construction of the Meixner process starts from an infinite divisible distribution with characteristic function

$$\varphi(u) = \left(\frac{\cos(b/2)}{\cosh((au - ib)/2)}\right)^{2d},$$

where $a > 0$, $-\pi < b < \pi$ and $d > 0$. This distribution characterizes the law of the process at one unit time and hence the Lévy exponent. The Meixner process does not have a Brownian component and the Lévy measure is absolutely continuous, hence its triplet is given by $(\mu, 0, \nu)$ where

$$\mu = ad\tanh(b/2) - 2d\int_1^{\infty}\frac{\sinh(bx/a)}{\sinh(\pi x/a)}dx$$

$$\nu(x) = d\frac{\exp(bx/a)}{x\sinh(\pi x/a)}. \tag{2}$$

We will make an abuse of notation by using the same name for the Lévy measure and its density if there is no confusion.

The β-family is a parametric family of Lévy processes introduced by Kuznetsov [2] which belongs to the more general family of processes called meromorphic Lévy processes (M-processes) introduced by Kuznetsov et al. [8]. A member of the $\boldsymbol{\beta}$-family is a 10-parameter process with triplet given by (μ, σ, ν) where the Lévy measure is absolutely continuous with density

$$\nu(x) = c_1\frac{e^{-\alpha_1\beta_1 x}}{(1 - e^{-\beta_1 x})^{\lambda_1}}1_{x>0} + c_2\frac{e^{\alpha_2\beta_2 x}}{(1 - e^{\beta_2 x})^{\lambda_2}}1_{x<0}, \tag{3}$$

where $\alpha_i > 0$, $\beta_i > 0$, $c_i \geq 0$ and $\lambda_i \in (0, 3)$. For the sake of completeness we reproduce here the expression of the characteristic exponent which is derived by Kuznetsov [2, Proposition 9] and satisfies

$$\Psi_{X_1}(z) = -i\mu z + \frac{\sigma^2}{2}z^2 - [c_1 I(z; \alpha_1, \beta_1, \lambda_1) + c_2 I(-z; \alpha_2, \beta_2, \lambda_2)], \tag{4}$$