



The numerical analysis on a Volterra equation with asymptotically periodic solution

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ABSTRACT

We consider order one operational quadrature methods on a certain integro-differential equation of Volterra type on $(0, \infty)$, with piecewise linear convolution kernels. The forms of discretization solution are patterned after a continuous one of Hannsgen (1979) [2]. An l^1 remainder stability and an error bound are derived.

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1. Introduction

This paper is concerned with the linear integro-differential equation

$$u'(t) + \int_0^t a(t-s)u(s) ds = 0, \quad t > 0, u(0) = 1, \quad (1.1)$$

($\prime = d/dt$), where u and $a(t)$ are real-valued functions. In addition, the kernel $a(t)$ has the piecewise linear form

$$a(t) = \sum_{l=1}^{\infty} \delta_l \left(1 - \frac{\min\{t, l\}}{l} \right), \quad (1.2)$$

with

$$\delta_l \geq 0, \quad 0 < a(0) = \sum_{l=1}^{\infty} \delta_l \equiv \delta < \infty, \quad (1.3)$$

and

$$\omega = \sqrt{\delta} = 2\pi j, \quad \text{for some integers } j. \quad (1.4)$$

It is easy to see that the kernel (1.2)–(1.4) is a special case in which

$$a(t) \text{ is nonnegative, nonincreasing, and convex on } (0, \infty) \text{ with } a \in L^1(0, 1) \text{ and } a(\infty) = 0. \quad (1.5)$$

Hannsgen showed in [1] that if (1.2)–(1.4) hold, then

$$u_1(t) \equiv u(t) - \frac{2}{\gamma} \cos \omega t \rightarrow 0 \quad (t \rightarrow \infty), \quad (1.6)$$

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with $\gamma = \frac{3\delta}{\delta} = 3$. Furthermore, in the same assumptions and adding an integrable condition

$$\int_0^\infty a(t) dt < \infty. \quad (1.7)$$

Hannsgen established in [2] that

$$\int_0^\infty (|u_1(t)| + |u'_1(t)|) dt < \infty, \quad (1.8)$$

and demonstrated that (1.8) need not hold if (1.7) fails to hold.

Our purpose in this paper is to study discretization of the problem (1.1). The methods considered will be based on the backward Euler approximation of the equation, combined with order one operational quadrature approximating the integral. The operational quadrature methods were introduced in [3] via an operational calculus for the Laplace transform. In order to describe the operational quadrature on the problem (1.1), we introduce time step denoted by k and a subscript n referring to the time level $t_n = nk$. We denote the approximation of $u(t_n)$ by u^n . The operational quadrature backward Euler scheme for approximating (1.1) is

$$\frac{u^n - u^{n-1}}{k} + \sum_{p=1}^n a_{n-p}(k)u^p = 0, \quad n \geq 1, \quad u^0 = 1, \quad (1.9)$$

where $a_p(k)$ are the coefficients of the power series

$$\widehat{a}\left(\frac{1-z}{k}\right) = \sum_{p=0}^\infty a_p(k)z^p, \quad (1.10)$$

and

$$\widehat{a}(s) = \int_0^\infty a(t)e^{-st} dt = \frac{\delta}{s} + \frac{1}{s^2} \sum_{l=1}^\infty \frac{\delta_l}{l} (e^{-ls} - 1) \quad (1.11)$$

is the Laplace transform of the kernel function a . The $\widehat{a}(s)$ is analytic for $\operatorname{Re} s > 0$ and continuous for $\operatorname{Re} s \geq 0$.

Multiplying (1.9) by z^n and summing from 1 to ∞ , we obtain for the generating function $\widetilde{u}(z) = \sum_{n=1}^\infty u^n z^n$, that

$$\widetilde{u}(z) = \frac{z}{k} \widehat{u}\left(\frac{1-z}{k}\right). \quad (1.12)$$

Moreover, we note that

$$\widehat{u}_1(s) = \frac{1}{s + \widehat{a}(s)} - \frac{2s}{\gamma(s^2 + \omega^2)}, \quad (\operatorname{Re} s > 0) \quad (1.13)$$

and $\widehat{u}_1(\bar{s}) = \overline{\widehat{u}_1(s)}$ ($\bar{s}$ = the complex conjugate of s). $\widehat{u}_1$ can be continuously extended to $\{\operatorname{Re} s \geq 0, s \neq \pm i\omega\}$.

For a discrete analogue of (1.6), we let

$$\bar{u}(z) = \frac{z}{k} \frac{2}{\gamma \left(\frac{1-z}{k}\right)^2 + \omega^2} = \bar{u}^1 z + \bar{u}^2 z^2 + \cdots + \bar{u}^n z^n + \cdots, \quad (1.14)$$

and

$$u_1^n = u^n - \bar{u}^n, \quad \text{for } n \geq 1. \quad (1.15)$$

Our first purpose is to show an l^1 remainder stability estimate, which is a discrete analogue of (1.8).

Theorem 1. Let (1.2)–(1.4) and (1.7) hold, and $u^n, \bar{u}^n$ are defined by (1.9) and (1.14), respectively. Then u_1^n satisfies

$$k \sum_{n=1}^\infty |u_1^n| \leq C, \quad (1.16)$$

where and after, C stands for a positive constant, independent of k and n , possibly with different values at different places.

Next, we shall show an l^1 remainder error estimate under the assumptions of Theorem 1, but adding a moment condition on the kernel $a(t)$

$$\int_0^\infty ta(t) dt < \infty. \quad (1.17)$$

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