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Delay dependent stability analysis of neutral systems with mixed time-varying delays and nonlinear perturbations ☆

R. Rakkiyappan, P. Balasubramaniam *, R. Krishnasamy

Department of Mathematics, Gandhigram Rural University, Gandhigram 624 302, Tamilnadu, India

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ABSTRACT

This paper is concerned with the problem of stability of neutral systems with interval time-varying delays and nonlinear perturbations. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties. A new delay-dependent stability condition is derived in terms of linear matrix inequality by constructing a new Lyapunov functional and using some integral inequalities without introducing any free-weighting matrices. Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods.

1. Introduction

It is well known that time-delay systems have been an active research area for the last few decades. The main reason is that time-delay frequently occurs in many practical systems, such as manufacturing systems, telecommunication and economic systems, and is a major cause of instability and poor performance. Generally, time-delay exists inevitably in control systems, which mainly results from the following: the time taken in the online data acquisition from sensors at different locations of the system; the time taken in the filtering and processing of the sensory data for the required control force to the actuator; the time taken by the actuator to produce the required control force, see for example [1] and references therein.

A neutral time-delay system contains delays both in its state, and in its derivatives of the state. Such a system can be found in population ecology [2], distributed networks containing lossless transmission lines [3], heat exchangers, robots in contact with rigid environments [4], etc. Because of its wider application, the problem of the stability of a delay-differential neutral system has received considerable attention by many researchers in the last two decades [5–30]. Neutral delay systems constitute a more general class than those of the retarded type. Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state. Especially, in the past few decades increased attention has been devoted to the problem of robust delay-independent stability or delay-dependent stability and stabilization via different approaches for linear neutral systems with delayed state and/or input and parameter uncertainties. Therefore, the problem of the stability and stabilization of neutral time-delay systems has attracted considerable attention during the past few years.

Using the Lyapunov–Razumikhin functional approach or the Lyapunov–Krasovskii functional approach several stability criteria have been proposed for delay-independent [11,12] and delay-dependent stability criteria [13–15] cases. Since delay-independent conditions are usually more conservative than the delay-dependent conditions, more attention has been paid

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* Corresponding author. Tel.: +91 451 2452371; fax: +91 451 2453071.
E-mail addresses: pbalgri@rediffmail.com, balugru@gmail.com (P. Balasubramaniam).

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to the study of delay-dependent conditions. For example, a delay-dependent stability criterion for uncertain neutral systems with time-varying discrete delay was obtained in [16] based on a model transformation and Park's inequality [31]. It is well known that nonlinearities, as time delays, may cause instability and poor performance of practical systems, which have driven many researchers to study the problem of nonlinear perturbed systems with state delays during recent years [17,32,33,18–23]. To the best of our knowledge, few results have been reported in the literature concerning the problem of robust stability of neutral systems with nonlinear perturbations and mixed time-varying neutral and discrete delays.

In this paper, we contribute to the further development of the stability analysis of neutral systems with nonlinear perturbations. The dynamical system under consideration consists of both time-varying neutral and discrete delays without any restriction on upper bounds of derivatives of time-varying delays. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. The proposed criterion is both neutral-delay dependent and discrete-delay dependent, and at the same time, is dependent on the derivative of the discrete and neutral delays. Therefore the methods in this paper are less conservative than those produced by previous approaches. Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 states the problem description and preliminaries. Section 3 includes the sufficient conditions for delay-dependent stability analysis of the system under consideration. Section 4 provides the delay-dependent robust stability criterion for the system. Section 5 provides illustrative examples and Section 6 concludes the paper.

2. Problem description and preliminaries

Consider the following neutral system with mixed time-varying delays and nonlinear perturbations:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + C\dot{x}(t-h(t)) + f_1(x(t),t) + f_2(x(t-\tau(t)),t) + f_3(\dot{x}(t-h(t)),t), \\ &x(\theta) = \phi(\theta), \qquad \dot{x}(\theta) = \varphi(\theta) \quad \forall\theta \in [-\max(\bar{h}, h_2), 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A, B, C \in \mathbb{R}^{n\times n}$ are constant matrices. $h(t), \tau(t)$ are neutral delay and time-varying discrete delay respectively, and they are assumed to satisfy

$$0 \le h(t) \le \bar{h}, \quad \dot{h}(t) \le h_d, \tag{2}$$

$$h_1 \le \tau(t) \le h_2, \quad \dot{\tau}(t) \le \mu, \tag{3}$$

where $\bar{h}, h_d, h_1, h_2$ and μ are constants. $\phi(\cdot), \varphi(\cdot)$ are the initial functions that are continuously differentiable on $[-\max(\bar{h}, h_2), 0]$. $f_1(x(t),t), f_2(x(t-\tau(t)),t), f_3(\dot{x}(t-h(t)),t)$ are unknown nonlinear perturbations satisfying $f_1(0,t) = 0, f_2(0,t) = 0, f_3(0,t) = 0$, and

$$\begin{aligned} &f_1^T(x(t),t)f_1(x(t),t) \le \alpha^2 x^T(t)x(t), \\ &f_2^T(x(t-\tau(t)),t)f_2(x(t-\tau(t)),t) \le \beta^2 x^T(t-\tau(t))x(t-\tau(t)), \\ &f_3^T(\dot{x}(t-h(t)),t)f_3(\dot{x}(t-h(t)),t) \le \gamma^2 \dot{x}^T(t-h(t))\dot{x}(t-h(t)), \end{aligned} \tag{4}$$

where $\alpha \ge 0$, $\beta \ge 0$ and $\gamma \ge 0$ are given constants, for simplicity, we denote $f_1 := f_1(x(t),t), f_2 := f_2(x(t-\tau(t)),t), f_3 := f_3(\dot{x}(t-h(t)),t)$.

Lemma 2.1 (*Schur Complement*). *Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, then*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0, \quad or, \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Lemma 2.2. *For any constant matrix $Z = Z^T > 0$ and scalars $\bar{h} > 0, h_1 > 0, h_2 > 0$ such that the following integrations are well defined:*

$$-\int_{t-\tau(t)}^{t} \rho^T(s)Z\rho(s)\mathrm{d}s \le -\frac{1}{h_2}\left(\int_{t-\tau(t)}^{t}\rho(s)\mathrm{d}s\right)^T Z \left(\int_{t-\tau(t)}^{t}\rho(s)\mathrm{d}s\right) \tag{5}$$

$$-\int_{t-\tau(t)}^{t-h_1} \rho^T(s)Z\rho(s)\mathrm{d}s \le -\frac{1}{(h_2-h_1)}\left(\int_{t-\tau(t)}^{t-h_1}\rho(s)\mathrm{d}s\right)^T Z \left(\int_{t-\tau(t)}^{t-h_1}\rho(s)\mathrm{d}s\right) \tag{6}$$