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Existence of solutions for nonlocal impulsive partial functional integrodifferential equations via fractional operators

Zuomao Yan

Department of Mathematics, Hexi University, Zhangye, Gansu 734000, PR China

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ABSTRACT

In this paper, by using the Leray–Schauder alternative, we have investigated the existence of mild solutions to first-order impulsive partial functional integrodifferential equations with nonlocal conditions in an α-norm. We assume that the linear part generates an analytic compact bounded semigroup, and that the nonlinear part is a Lipschitz continuous function with respect to the fractional power norm of the linear part. An example is also given to illustrate our main results.

1. Introduction

The theory of impulsive differential equations has found wide applications in many branches of physics and technical sciences; see the monographs of Lakshmikantham et al. [1], Bainov and Simeonov [2], Benchohra et al. [3], and the papers of Rogovchenko [4] and the survey papers of Rogovchenko [5], Bainov [6] and the references therein. Recently, much attention has been paid to existence results for the impulsive differential and integrodifferential equations in abstract spaces; for example, see [7–13]. In this paper, we are concerned with the following impulsive partial functional integrodifferential equations with nonlocal conditions

$$x'(t) = Ax(t) + F\left(t, x(\sigma_1(t)), \ldots, x(\sigma_n(t)), \int_0^t h(t, s, x(\sigma_{n+1}(s)))\mathrm{d}s\right),$$

$$t \in J = [0, b],\ t \neq t_k,\ k = 1, \ldots, m, \tag{1.1}$$

$$x(0) + g(x) = x_0, \tag{1.2}$$

$$\Delta x(t_k) = I_k(x(t_k)), \quad k = 1, \ldots, m, \tag{1.3}$$

where the unknown $x(\cdot)$ takes values in the Banach space X, and A is the infinitesimal generator of a compact, analytic semigroup $T(t)$, $t > 0$; $0 < t_1 < \cdots < t_m < b$, are prefixed points and the symbol $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, where $x(t_k^-)$ and $x(t_k^+)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively. F, h, g, I_k and σ_i, $i = 1, \ldots, n+1$, are given functions to be specified later.

Nonlocal conditions were initiated in [14,15] when he proved the existence and uniqueness of mild and classical solutions of nonlocal Cauchy problems. As remarked in [15,16], the nonlocal condition can be more useful than the standard initial

E-mail address: yanzuomao@163.com.

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condition to describe some physical phenomena. For other contributions on the nonlocal problems; see [17–23] and the references therein. Very recently, there has been extensive study of impulsive differential equations with nonlocal conditions, and concerning this matter we cite the pioneer works; Liang et al. [24], and Anguraj and Karthikeyan [25] have studied the existence, uniqueness and continuous dependence of a mild solution of a nonlocal Cauchy problem for an impulsive neutral functional differential evolution equation. The purpose of this paper is to continue the study of these authors. We get the existence results for mild solutions of problem (1.1)–(1.3) with an α-norm as in [19], assuming that F is defined on $J \times X_\alpha^{n+1}$, the nonlocal item g only depends upon the continuous properties on $C(J, X_\alpha)$, where $X_\alpha = D(A^\alpha)$, for some $0 < \alpha < 1$, the domain of the fractional power of A. Our results are based on the analytic semigroup theory of linear operators, the Banach contraction principle and the Leray–Schauder alternative.

The rest of this paper is organized as follows: In Section 2 we recall briefly some basic definitions and preliminary facts which will be used throughout this paper. The existence theorems for problem (1.1)–(1.3) and their proofs are arranged in Section 3. Finally, in Section 4 an example is presented to illustrate the applications of the obtained result.

2. Preliminaries

In this section, we shall introduce some notations, definitions and lemmas which are used throughout this paper.

Let $(X, \|\cdot\|)$ be a Banach space. $C(J, X)$ is the Banach space of continuous functions from J into X with the norm

$$\|x\|_J = \sup\{\|x(t)\| : t \in J\}$$

and let $L(X)$ denote the Banach space of bounded linear operators from X to X. A measurable function $x : J \to X$ is Bochner integrable if and only if $\|x\|$ is Lebesgue integrable (for properties of the Bochner integral; see [26]). $L^1(J, X)$ denotes the Banach space of measurable functions $x : J \to X$ which are Bochner integrable normed by

$$\|x\|_{L^1} = \int_0^b \|x(t)\| \mathrm{d}t \quad \text{for all } x \in L^1(J, X).$$

The notation $B_r[x, X]$ stands for the closed ball with center at x and radius $r > 0$ in X.

Throughout this paper, $A : D(A) \to X$ is the infinitesimal generator of a compact analytic semigroup of uniformly bounded linear operators $T(t)$. Let $0 \in \rho(A)$. Then it is possible to define the fractional power A^α, for $0 < \alpha \le 1$, as a closed linear operator on its domain $D(A^\alpha)$ (see [27]). Furthermore, the subspace $D(A^\alpha)$ is dense in X and the expression

$$\|x\|_\alpha = \|A^\alpha x\|, \quad x \in D(A^\alpha),$$

defines a norm on $D(A^\alpha)$. Let X_α be the Banach space $D(A^\alpha)$ endowed with the norm $\|x\|_\alpha$, and in the following, we use $\|\cdot\|_\alpha$ to denote the operator norm in X_α. Then for each $0 < \alpha \le 1$, X_α is a Banach space, and $X_\alpha \hookrightarrow X_\beta$ for $0 < \beta < \alpha \le 1$ and the imbedding is compact whenever the resolvent operator of A is compact. For semigroup $\{T(t)_{t\ge 0}\}$, the following properties will be used:

(a) there is an $M \ge 1$ such that $\|T(t)\| \le M$, for all $0 \le t \le b$;
(b) for any $0 \le \alpha \le 1$, there exists a constant $M_\alpha > 0$ such that

$$\|A^\alpha T(t)\| \le \frac{M_\alpha}{t^\alpha}, \quad 0 < t \le b.$$

In order to define the solution of (1.1)–(1.3), we introduce the space $PC([0, b], X_\alpha) = \{x : J \to X_\alpha : x(t)$ is continuous at $t \ne t_k$ and left continuous at $t = t_k$ and the right limit $x(t_k^+)$ exists for $k = 1, 2, \dots, m\}$, which is a Banach space with the norm

$$\|x\|_{PC} := \sup_{t\in J} \|x(t)\|_\alpha.$$

Then $PC(J, X_\alpha)$ is a Banach space.

To simplify the notations, we put $t_0 = 0,\ t_{m+1} = b$ and for $x \in PC([0, b], X_\alpha)$ we denote by $\hat{x}_k \in C([t_k, t_{k+1}]; X_\alpha),\ k = 0, 1, \dots, m$, the function given by

$$\hat{x}_k(t) := \begin{cases} x(t) & \text{for } t \in (t_k, t_{k+1}], \\ x(t_k^+) & \text{for } t = t_k. \end{cases}$$

Moreover, for $B \subseteq PC([0, b], X_\alpha)$ we denote by $\hat{B}_k,\ k = 0, 1, \dots, m$, the set $\hat{B}_k = \{\hat{x}_k : x \in B\}$.

Definition 2.1. A function $x(\cdot) \in PC(J, X_\alpha)$ is said to be a mild solution to problem (1.1)–(1.3) if it satisfies the following integral equation

$$x(t) = T(t)[x_0 - g(x)] + \sum_{0<t_k<t} T(t - t_k)I_k(x(t_k)) + \int_0^t T(t - s)F\left(s, x(\sigma_1(s)), \dots, x(\sigma_n(s)), \int_0^s h(s, \tau, x(\sigma_{n+1}(\tau)))\mathrm{d}\tau\right) \mathrm{d}s, \quad 0 \le t \le b. \tag{2.1}$$