Journal of Computational and Applied Mathematics 235 (2011) 5307–5325

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

On pricing arithmetic average reset options with multiple reset dates in a lattice framework

Massimo Costabile, Ivar Massabó, Emilio Russo *

Department of Business Administration, University of Calabria, Ponte Bucci cubo 3 C, 87036, Rende (CS), Italy

ARTICLE INFO

Article history:
Received 20 November 2009
Received in revised form 19 May 2011

Keywords:
Reset options
Binomial algorithms
Discrete time models

ABSTRACT

We develop a straightforward algorithm to price arithmetic average reset options with multiple reset dates in a Cox et al. (CRR) (1979) [10] framework. The use of a lattice approach is due to its adaptability and flexibility in managing arithmetic average reset options, as already evidenced by Kim et al. (2003) [9]. Their model is based on the Hull and White (1993) [5] bucketing algorithm and uses an exogenous exponential function to manage the averaging feature, but their choice of fictitious values does not guarantee the algorithm's convergence (cfr., Forsyth et al. (2002) [11]). We propose to overcome this drawback by selecting a limited number of trajectories among the ones reaching each node of the lattice, where we compute effective averages. In this way, the computational cost of the pricing problem is reduced, and the convergence of the discrete time model to the corresponding continuous time one is guaranteed.

1. Introduction

The increasing popularity of Asian-style options in financial markets makes their pricing problem a hot topic that is being studied by many authors looking for developing efficient pricing models. Indeed, the averaging feature is often desirable since it mitigates the sensitivity of the contract payoff to large price movements.

We propose an algorithm for pricing arithmetic average reset options characterized by multiple reset dates. The payoff of these securities is similar to that of plain-vanilla options, but, in addition, at some pre-determined reset dates they allow to update the strike price with the value of the arithmetic average of the asset prices registered during fixed monitoring windows. This means that the strike price of such options is stochastic depending upon the arithmetic average.

To illustrate the update of the strike price, we refer to a European reset call option, characterized by $t_1, \dots, t_N$ reset dates, and time to maturity T, written on an underlying asset with value S_t at time t and dynamics described by a geometric Brownian motion. The option time to maturity is divided into $N+1$ monitoring windows for which, without loss of generality, we may suppose a constant length $t_l - t_{l-1} = \frac{T}{N+1}$, with $l = 1, \dots, N+1$, $t_0 = 0$ and $t_{N+1} = T$. Let us denote by A_l, $l = 1, \dots, N$, the arithmetic average of the underlying asset prices registered during the lth monitoring window, i.e., in time period $(t_{l-1}, t_l]$, by K_0 the initial strike price used in the first monitoring window, and by $K_l = \min(K_{l-1}, A_l)$, $l = 1, \dots, N$, the strike price[1] used in the $(l+1)$th monitoring window $[t_l, t_{l+1})$. At the end of the Nth monitoring window, it lies at the last reset date, t_N, where the strike price is finally modified according to $K_N = \min(K_{N-1}, A_N)$ and remains unchanged up to maturity. Indeed, after the Nth reset date (in the interval $(t_N, T]$), the reset call option becomes a standard

* Corresponding author. Tel.: +39 0984493387; fax: +39 0984492277.
E-mail addresses: massimo.costabile@unical.it (M. Costabile), i.massabo@unical.it (I. Massabó), emilio.russo@unical.it (E. Russo).

[1] In the case of a put option with $t_1, \dots, t_N$ reset dates, the strike price is reset according to $K_l = \max(K_{l-1}, A_l)$, $l = 1, \dots, N$, where K_0 is the strike price fixed at inception.

0377-0427/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2011.05.041

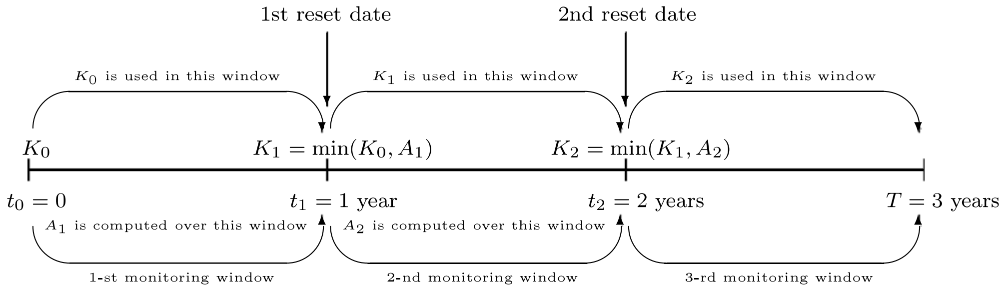

Fig. 1. The initial strike price is K_0. Average A_1 is computed considering the underlying asset prices attained in the 1st monitoring window, and the strike price is reset at level K_1 at the 1st reset date. Average A_2 is computed upon the asset prices registered in the 2nd monitoring window and the strike price is reset at level K_2 at the 2nd reset date. The latter strike price is used up to maturity.

call one with strike price $K_N = \min(K_0, A_1, \ldots, A_N)$, i.e., the payoff at maturity is $\max(S_T - K_N, 0)$. In Fig. 1, we present an example of a European arithmetic average reset call option with lifetime $T = 3$ years characterized by two equally spaced reset dates.

Contrary to the case of European options which reset the strike price to the current asset value[2] or to a pre-specified function of the asset prices with known distribution,[3] the pricing problem of arithmetic average reset options must be tackled with numerical approximation methods because their payoff distribution is not known, even in the simple log-normal framework. Among them, lattice-based methodologies represent a valuable resource for their flexibility and efficiency in managing path-dependent options; they are really appreciated by practitioners for their simplicity in implementation.

In a lattice framework, the main obstacle to price reset options with payoff depending upon the arithmetic average of the underlying asset prices is relative to the fact that the number of alternative arithmetic averages which can be realized at a node grows very fast with the number of time steps. To handle this complexity, a lattice-based model has been proposed in [9] who adapt the forward shooting grid method for valuing path-dependent options developed in [5] by adding two augmented state variables to the standard Cox et al. [10] (CRR) model. The first one, used to generate a set of representative strike prices at each node of the lattice, has the form K_0e^{-kh}, where K_0 is the initial strike price, h is a positive real number, and k ranges between 0 and a suitable integer so that the set contains the possible minimum strike price at that node. The second state variable associates with each node a set of representative averages computed as $A_{\max}e^{-mh}$, where $A_{\max}$ is the maximum possible value of the arithmetic average at the considered node and m ranges between 0 and an integer m^* which allows the minimum average $A_{\min}$ to be included into the set. These exponential functions produce fictitious values both for the strike price and the arithmetic average which make the performance of the model strongly dependent upon the value assumed by the parameter h and the interpolation technique chosen. In order to keep the algorithm computationally efficient, they choose h proportional to $\sigma\sqrt{\Delta t}$ (where σ is the underlying asset return volatility, while Δt is the step length) but this choice does not assure the convergence of their discrete time model to the corresponding continuous time one. Indeed, as evidenced in [11], to achieve convergence in a forward shooting grid model when a linear interpolation procedure is applied, h must be chosen proportional to the step length of the lattice, Δt.

The essence of our approach is to overcome the drawback of the Kim et al. algorithm by choosing sets of representative averages made up of effective values computed upon selected actual paths reaching each node of the lattice. Contrary to Kim et al., the proposed procedure does not rely upon any external parameter. At the declared reset dates, these averages are used to eventually update the strike price values. The backward recursion scheme, coupled with linear interpolation, furnishes a way to compute the option price at inception. The result is a model which reduces the computational complexity of the option evaluation problem and assures the convergence to the corresponding continuous time model (the convergence analysis is carried out in Appendix A). Finally, one of the main feature of the model is its flexibility which allows an immediate application to determine the option replicating portfolio and accommodates early exercise for valuing American options.

The rest of the paper is organized as follows. In Section 2, we present the binomial model for pricing arithmetic average reset options with multiple reset dates. Section 3 is devoted to illustrate the performance of our algorithm. Section 4 concludes with a summary.

[2] To cite a few, Gray and Whaley [1,2] derive a valuation formula for the case of one reset date while Cheng and Zhang [3] present a closed form pricing formula generalized to the case of multiple reset dates. Furthermore, Kwok and Lau [4] propose a lattice model based on the algorithm developed in [5] for pricing path-dependent options. Models for options with different reset features or developed in different frameworks have been also proposed. Among others, we recall the recent contribution of Li et al. [6], which works in a stochastic interest rate framework, and of Yu and Shaw [7] who consider the case of an option with a snapshot reset feature.

[3] Among others, for geometric average reset options, Cheng and Zhang [3] provide an explicit formula in the case of one reset date while Dai et al. [8] derive an analytic formula in the case of multiple monitoring windows as a corollary of a general formula used for pricing a large class of path-dependent options.