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An explicit method for systems of equilibrium problems and fixed points of infinite family of nonexpansive mappings[☆]

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ABSTRACT

Let H be a Hilbert space, $\{T_i\}_{i\in\mathbb{N}}$ a family of nonexpansive mappings from H into itself, $G_i : C \times C \to \mathbb{R}$ a finite family of equilibrium functions ($i \in \{1, 2, \ldots, K\}$), A a strongly positive bounded linear operator with coefficient $\bar{\gamma}$ and f an α-contraction on H. Let W_n be the mapping generated by $\{T_i\}$ and $\{\lambda_n\}$ as in (1.5), let $S^k_{r_{k,n}}$ be the resolvent generated by G_k and $r_{k,n}$ as in Lemma 2.4. Moreover, let $\{r_{k,n}\}_{k=1}^K$, $\{\epsilon_n\}$ and $\{\lambda_n\}$ satisfy appropriate conditions and $F := (\bigcap_{k=1}^K SEP(G_k)) \cap (\bigcap_{n\in\mathbb{N}} Fix(T_n)) \neq \emptyset$; we introduce an explicit scheme which defines a suitable sequence as follows:

$$z_{n+1} = \epsilon_n \gamma f(z_n) + (I - \epsilon_n A) W_n S^1_{r_{1,n}} S^2_{r_{2,n}} \cdots S^K_{r_{K,n}} z_n \quad \forall n \in \mathbb{N}$$

and $\{z_n\}$ strongly converges to $x^* \in F$ which satisfies the variational inequality $\langle (A - \gamma f)x^*, x - x^* \rangle \geq 0$ for all $x \in F$. The results presented in this paper mainly extend and improve some recent results in [Vittorio Colao, et al., An implicit method for finding common solutions of variational inequalities and systems of equilibrium problems and fixed points of infinite family of nonexpansive mappings, Nonlinear Anal. 71 (2009) 2708–2715; S. Plubtieng, R. Punpaeng, A general iterative method for equilibrium problems and fixed point problems in Hilbert spaces, J. Math. Anal. Appl. 336 (2007) 455–469; S. Takahashi, W. Takahashi, Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces, J. Math. Anal. Appl. 331 (2007) 506–515].

1. Introduction

Throughout this paper, we always assume that H is a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, respectively, C is a nonempty closed convex subset of H and P_C is the metric projection of H onto C. In the following, we denote by "$\to$" strong convergence, by "$\rightharpoonup$" weak convergence and by "$\mathbb{R}$" the real number set.

Let $G : C \times C \to \mathbb{R}$ be an equilibrium function, that is

$$G(u, u) = 0 \quad \text{for every } u \in C.$$

The equilibrium problem is defined as follows:

$$\text{Find } \tilde{x} \in C \text{ such that } G(\tilde{x}, y) \geq 0 \ \text{ for all } y \in C. \tag{1.1}$$

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A solution of (1.1) is said to be an equilibrium point and the set of the equilibrium points is denoted by $SEP(G)$. This topic has been considered in [1,2]. We shall assume some mild conditions on G in such a way that results can be applied in several cases including optimization problems, fixed point problems and convex vector minimization problems [3–6].

For solving the equilibrium problem on a bifunction $G : C \times C \to \mathbb{R}$, let us assume that G satisfies the following conditions:

(A_1) $G(x, x) = 0$ for all $x \in C$;
(A_2) G is monotone, i.e., $G(x, y) + G(y, x) \le 0$ for all $x, y \in C$;
(A_3) for each $x, y, z \in C$, $\lim_{t\to 0} G(tz + (1 - t)x, y) \le G(x, y)$;
(A_4) for each $x \in C$, $y \mapsto G(x, y)$ is convex and lower semicontinuous.

Let A be a bounded linear operator on H, a mapping $f : H \to H$ an α-contraction (i.e. $\|f(x)-f(y)\| \le \alpha\|x-y\|$, $\forall x, y \in H$). The following variational inequality problem with viscosity is of great interest [7,8]. Find x^* in C such that

$$\langle (A - \gamma f)x^*, x - x^*\rangle \ge 0, \quad \forall x \in C \tag{1.2}$$

which is the optimality condition for the minimization problem

$$\min_{x\in C} \frac{1}{2}\langle Ax, x\rangle - h(x),$$

where h is a potential function for γf (i.e., $h'(x) = \gamma f(x)$ for $x \in H$).

On the other hand, given a nonexpansive map T, from H into itself (i.e. $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in H$) and using $Fix(T) := \{x \in H : Tx = x\}$ denote the fixed point set of T, finding an optimal point in $Fix(T)$ is a matter of interest in various branches of science (see [9–11]).

Recently, Plubtieng and Punpaeng [12] proved a strong convergence theorem for an implicit iterative sequence $\{x_n\}$ obtained from the viscosity approximation iteration method (1.3) for finding a common element in $SEP(G) \cap Fix(T)$:

$$\begin{cases} G(y_n, u) + \dfrac{1}{r_n}\langle u - y_n, y_n - x_n\rangle \ge 0, \quad \forall u \in C, \\ x_n = \alpha_n f(x_n) + (1 - \alpha_n)Ty_n. \end{cases} \tag{1.3}$$

And recently, S. Takahashi and W. Takahashi [13] introduced the following explicit iterative scheme (1.4)

$$\begin{cases} G(y_n, u) + \dfrac{1}{r_n}\langle u - y_n, y_n - x_n\rangle \ge 0, \quad \forall u \in C, \\ x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)Ty_n \end{cases} \tag{1.4}$$

and also proved that the sequence $\{x_n\}$ defined by (1.4) strongly converges to a common element of $SEP(G) \cap Fix(T)$.

Let C be a nonempty convex subset of a Banach space X. Let $\{T_i\}$ be an infinite family of nonexpansive mappings of C into itself and let $\{\lambda_i\}$ be a real sequence such that $0 \le \lambda_i \le 1$ for every $i \in \mathbb{N}$. Following [14], for any $n \ge 1$, we define a mapping W_n of C into itself as follows:

$$\begin{aligned} &U_{n,n+1} := I, \\ &U_{n,n} := \lambda_n T_n U_{n,n+1} + (1 - \lambda_n)I, \\ &\vdots \\ &U_{n,k} := \lambda_k T_k U_{n,k+1} + (1 - \lambda_k)I, \\ &\vdots \\ &U_{n,2} := \lambda_2 T_2 U_{n,3} + (1 - \lambda_2)I, \\ &W_n := U_{n,1} = \lambda_1 T_1 U_{n,2} + (1 - \lambda_1)I. \end{aligned} \tag{1.5}$$

Very recently, Colao [15] studied the following implicit iterative sequence $\{z_n\}$ defined by (1.6), with the initial guess $z_0 \in H$ chosen arbitrarily and satisfying appropriate conditions,

$$z_n = \epsilon_n \gamma f(z_n) + (I - \epsilon_n A)W_n S^1_{r_{1,n}} S^2_{r_{2,n}} \cdots S^k_{r_{k,n}} z_n \quad \forall n \in \mathbb{N} \tag{1.6}$$

and proved that the sequence $\{z_n\}$ converges strongly to $x^* \in F := (\bigcap_{k=1}^{K} SEP(G_k)) \cap (\bigcap_{n\in\mathbb{N}} Fix(T_n))$ which also satisfies the variational inequality (1.2).

In this paper, motivated in [15,12,13], we study an explicit approximation process as follows:

$$z_{n+1} = \epsilon_n \gamma f(z_n) + (I - \epsilon_n A)W_n S^1_{r_{1,n}} S^2_{r_{2,n}} \cdots S^k_{r_{k,n}} z_n \quad \forall n \in \mathbb{N}. \tag{1.7}$$

2. Preliminaries

In a real Hilbert space H, the following inequality holds

$$\|x + y\|^2 \le \|x\|^2 + 2\langle y, x + y\rangle, \quad \text{for all } x, y \in H. \tag{2.1}$$