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The use of QP-free algorithm in the limit analysis of slope stability

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ABSTRACT

In many mountainous areas, landslides and slope instabilities frequently occur after heavy rainfall and earthquake, and result in enormous casualties and huge economic losses. In order to mitigate the landslides hazard efficiently, a method is required for a better understanding of stability analysis. Fortunately, upper bound theorem of limit analysis provides a practical and effective upper bound approach to evaluate the stability of slopes. And in this approach, the search for the minimum factor of safety can be formulated as a nonlinear constrained optimization. In general, the SQP-type algorithms are used to solve this optimization problem. However, it is quite time consuming and difficult to search the optimum from an arbitrary starting point based on the SQP-type algorithms. Fortunately, a QP-free algorithm based on penalty function and active-set strategy can be globally convergent toward the KKT points with arbitrary starting point, and the rate of convergence is local superlinear or even quadratic. Two classical problems of slope stability are solved by this QP-free algorithm. The results show that the QP-free algorithm would be the better choice than SQP-type algorithms for solving the nonlinear constrained optimization problem which is derived from the upper bound limit analysis of slope stability.

1. Introduction

Landslides, defined as the movement of mass of rock, debris or earth down a slope, have caused large number of casualties and huge economic losses in mountainous areas of the world, especially, China Mainland, Tai Wan, and Hong Kong. As a result of slope instability, landslides can be triggered by a variety of external stimulus, such as heavy rainfall or earthquake shaking that cause a rapid increase in shear stress or decrease in shear strength of slope-forming materials. In order to mitigate the landslides hazard efficiently, the first step of the slope stability analysis is to develop a practical and effective approach. Various researchers such as Drucker [1], Chen [2], Sloan [3], Sloan and Kleeman [4], Lyamin and Sloan [5] dedicate to develop the upper bound approach based on the upper bound theorem of limit analysis. Recently, Chen et al. [6–9] have presented a much simpler and straightforward upper bound approach based on Rigid Finite Element Method (RFEM). In this approach, the problem of finding the minimum value of factor of safety can be formulated as a nonlinear programming (NLP) problem.

1.1. Upper bound approach based on RFEM

In the rigid finite-element-based upper bound approach of slope stability analysis, the slope stability is evaluated by means of the safety factor *F* which has the same definition as in the Limit Equilibrium Method (LEM) [10]. In general, it can

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be defined as follows:

$$c'_r = \frac{c'}{F}; \qquad \tan\phi'_r = \frac{\tan\phi'}{F} \tag{1}$$

where c' and ϕ' are the effective cohesion and friction angle, respectively.

In the numerical formulation of upper bound approach based on RFEM, the sliding soil mass is divided into a proper number of rigid finite elements connected by the interfaces, and then, a kinematically admissible velocity field is constructed. The kinematically admissible velocity discontinuities should satisfy the constraints of Mohr–Coulomb yield criterion, associated flow rule, energy–work balance equation and prescribed velocity boundary conditions. Finally, the model of finding the minimum of safety factor F can be formulated as a nonlinear constrained optimization problem (NLP) [6–9]:

$$\begin{aligned} \min \quad & F \\ \text{s.t.} \quad & \begin{cases} A_1 V_G - A_2 V_d = 0 \\ A_3 V_g - \dfrac{1}{F} A_4 V_d = 0 \\ A_5 V_G - \dfrac{1}{F} A_6 V_d = 0 \\ V_g^k - \bar{V} = 0 \\ V_d \geq 0. \end{cases} \end{aligned} \tag{2}$$

The explanation of the notations can be found in Refs. [6–9]

1.2. The optimization algorithms for NLP

In the upper bound limit analysis of slope stability, a key aspect is the efficiency of NLP (2). Linear programming (LP) has been used in [3,4]. However, in their methods the general yield function need to be replaced by numerous linear inequality constraints, and it means that the computational cost becomes prohibitive for the large-scale problems. With the development of NLP theory, Lyamin and Sloan [5] apply the NLP algorithm based on feasible direction to obtain strict upper bound with a native form of yield function.

As to the rigid finite-element-based upper bound approach of slope stability analysis, Chen et al. [9] applied the FSQP algorithm based on the CFSQP software package [11] to obtain solutions for such nonlinear optimization problems. However, Baker [12] pointed out that the solution procedure of the proposed approach in [6–9] is not efficient compared to the conventional Stress Reduction Method (SRM). Chen et al.'s reply admitted this statement, and ascribed the inefficiency to FSQP algorithms of nonlinear programming [12]. Indeed, FSQP algorithms still require solving more QP subproblems to keep the feasibility of iteration points, which is computationally expensive. For the sake of reducing the computational cost, A QP-free algorithm was first proposed in [13], and further study on QP-free algorithm can be found in [14–19]. The above FSQP and QP-free algorithms all require that the starting points should be feasible, and the iteration points generated by the algorithms satisfy the inequality constraints. However, it is very difficult or even impossible to obtain a feasible starting point when the number of rigid finite element becomes very large. In addition, it will cost a large amount of computation to obtain a feasible iteration point, especially for some large-scale problems. So, the algorithm which is used to solve the nonlinear constrained optimization problem derived from the upper bound limit analysis based on rigid finite element method should keep two conditions as follows.

(1) The algorithm can solve this problem with an arbitrary starting point, and need not keep the feasibility of iteration points.
(2) The algorithm should be a completely QP-free method.

Fortunately, Gao et al. [15] proposed a QP-free algorithm based on a ε-active set and a special penalty function as the merit function; this algorithm can solve the nonlinear constrained optimization with arbitrary starting point. By relaxing the limitation of feasibility of the iteration point, it could reduce the cost of computation and be suitable for solving the upper bound limit analysis of slope stability problems. Therefore, we applied this QP-free algorithm to obtain solutions for such nonlinear optimization problems.

2. Slope stability analysis based on QP-free algorithm

In the NLP (2) proposed in [6–9], the safety factor F is the objective function, and it is taken into account in the constraints simultaneously. In fact, the safety factor can be expressed in terms of kinematically admissible velocity field and its jump. In this section, we first reformulated the NLP of upper bound approach based on RFEM into a standard form.