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Numerical solution for the weakly singular Fredholm integro-differential equations using Legendre multiwavelets

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ABSTRACT

An effective method based upon Legendre multiwavelets is proposed for the solution of Fredholm weakly singular integro-differential equations. The properties of Legendre multiwavelets are first given and their operational matrices of integral are constructed. These wavelets are utilized to reduce the solution of the given integro-differential equation to the solution of a sparse linear system of algebraic equations. In order to save memory requirement and computational time, a threshold procedure is applied to obtain the solution to this system of algebraic equations. Through numerical examples, performance of the present method is investigated concerning the convergence and the sparseness of the resulted matrix equation.

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1. Introduction

The main aim of the current paper is to present high order numerical methods for solving the weakly singular Fredholm integro-differential equations of the form

$$\mu_1 u'(t) + \mu_2 u(t) + \int_a^b k(s,t)|t-s|^{-\alpha} u(s)\mathrm{d}s = f(t), \quad a \le t \le b, \tag{1.1}$$

with

$$u(a) = \lambda, \tag{1.2}$$

where $\mu_1.\mu_2 \neq 0,\ 0 < \alpha < 1$ and λ is a constant and k and f are given functions and u is the solution to be determined. Moreover we assume that the kernel k is in $L^2[a,b]^2$ and the unknown u and the right-hand-side f are in $L^2[a,b]$. For simplicity, we restrict our attention to the interval $[a,b] = [0,1]$. Also we suppose that $k(s,t)$ satisfies in the Lipschitz condition,

$$|k(s_1,t) - k(s_2,t)| \le L_s|s_1 - s_2|, \tag{1.3}$$

where L_s is the Lipschitz constant.

Integral equation (1.1) with $\mu_1 = 0$ often arises in practical applications such as Dirichlet problems, mathematical problems of radiative equilibrium and radiative heat transfer problems [1–3].

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As said in [4] the construction of high order methods for these equations is, however, not an easy task because of the singularity in the "weakly singular" kernel $|t-s|^{-\alpha}k(t,s)$; in fact, in this case the solution u is generally not differentiable at the endpoints (i.e. $t=a$ and $t=b$) [5,4,6,2,7], and due to this, to the best of the authors' knowledge the best convergence rate if achieved remains only at polynomial order. For example, if we set the uniform meshes with $n+1$ grid points and apply the spline methods of order m, then the convergence rate is $O(n^{-2p})$ at most [8,9,3], and it cannot be improved by increasing m. We refer the interested reader to [4] for a nice investigation on weakly singular Fredholm integral equations of the second kind.

In the current paper we present an alternative way of solving weakly singular Fredholm integral equations which is based on using the Legendre multiwavelets.

Wavelet theory is relatively new and is an emerging area in mathematical research. In recent years, wavelets have found their way into different fields of science and engineering. Wavelet analysis assumed significance due to the successful applications in signal and image processing during the eighties. The smooth orthonormal basis obtained by the translation and dilation of a single function in a hierarchical fashion proved very useful to develop compression algorithms for signals and images up to a chosen threshold of relevant amplitudes [10,11].

After discretizing the given differential equation in a conventional way like the finite difference approximation [12–14], wavelets can be used for algebraic manipulations in the obtained system of equations which may lead to better condition number of the resulting system. Beylkin's approach [15] to the study of partial differential equations is based on this idea where he has shown that the differential operator is approximately diagonal in the wavelet bases.

Another approach to the study of differential equations is to use the wavelet bases in the place of other conventional bases like Fourier, Legendre or Chebyshev bases in the spectral methods [16,17,15,18–22] and so on. Wavelets permit the accurate representation of a variety of functions and operators. Orthogonal functions and polynomial series have been receiving considerable attention in dealing with various problems of dynamic systems. In the current investigation, we consider the application of wavelet bases in the solution of integro-differential equations. The conventional discretization of the integral equations leads to a dense matrix owing to the non-local nature of the situation. It was discovered in [23] that the representation of an integral operator by compactly supported orthonormal wavelets produces sparse matrices to some degrees of precision. Alpert et al. [16] introduced wavelet like bases for the fast solution [24] of the second kind integral equations in $O(n\log n)$ operations where n is the number of points in the discretization. Glowinski et al. [25] considered wavelet based on variational methods to solve the one-dimensional linear and nonlinear ordinary differential equations.

Different variations of wavelet bases (orthogonal, biorthogonal, multiwavelets) have been presented and the design of the corresponding wavelet and scaling functions have been addressed [26,10,11,27]. Multiwavelets are generated by more than one scaling function [28,27]. Multiwavelets have some advantages in comparison to single wavelets. The properties such as short support, orthogonality, symmetry and vanishing moments are known to be important in signal processing and numerical methods. A single wavelet cannot possess all these properties at the same time. On the other hand, a multiwavelet system can have all of them simultaneously [29]. This suggests that multiwavelets could perform better in various applications.

In this paper, we use the Legendre multiwavelets for solving weakly singular integro-differential equations. These multiwavelets have been constructed in [30] and also have been considered in [17,31,29]. Our method consists of reducing the given weakly singular integro-differential equation to a set of algebraic equations by expanding the equation as Legendre multiwavelets with unknown coefficients. The properties of these multiwavelets are then utilized to evaluate the unknown coefficients. We refer the interested reader to [32–44] for some basic ideas on numerical solution of integral equations. Also the reader can see [45,46] for applications of the partial integro-differential equations.

The paper is organized as follows: Section 2 is devoted to the basic formulation of the Legendre multiwavelets required for our subsequent development. In Section 3 the proposed method is used to approximate the weakly singular integro-differential equations. In Section 4, we report our numerical findings and demonstrate the accuracy of the proposed numerical scheme by considering several examples.

2. Legendre multiwavelet systems

2.1. Multiresolution analysis

For functions $\phi^m \in L^2(R),\ m=0,\dots,r$, let a reference subspace or sample space V_0 be generated as the L^2-closure of the linear span of the integer translation of ϕ^m, namely:

$$V_0 = \operatorname{clos}_{L^2}\left\langle \phi^m(.-k) : k\in Z\right\rangle, \quad m=0,\dots,r,$$

and consider other subspace

$$V_j = \operatorname{clos}_{L^2}\left\langle \phi^m_{j,k} : k\in Z\right\rangle, \quad j\in Z, m=0,\dots,r,$$

where $\phi^m_{j,k} = \phi^m(2^j x-k), j,k\in Z,\ m=0,\dots,r$.

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