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Methods of critical value reduction for type-2 fuzzy variables and their applications

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ABSTRACT

A type-2 fuzzy variable is a map from a fuzzy possibility space to the real number space; it is an appropriate tool for describing type-2 fuzziness. This paper first presents three kinds of critical values (CVs) for a regular fuzzy variable (RFV), and proposes three novel methods of reduction for a type-2 fuzzy variable. Secondly, this paper applies the reduction methods to data envelopment analysis (DEA) models with type-2 fuzzy inputs and outputs, and develops a new class of generalized credibility DEA models. According to the properties of generalized credibility, when the inputs and outputs are mutually independent type-2 triangular fuzzy variables, we can turn the proposed fuzzy DEA model into its equivalent parametric programming problem, in which the parameters can be used to characterize the degree of uncertainty about type-2 fuzziness. For any given parameters, the parametric programming model becomes a linear programming one that can be solved using standard optimization solvers. Finally, one numerical example is provided to illustrate the modeling idea and the efficiency of the proposed DEA model.

1. Introduction

The concept of a type-2 fuzzy set was first proposed in [1] as an extension of an ordinary fuzzy set. Since then, many researchers have employed the theory in their studies. For example, Mitchell [2] used the concept of an embedded type-1 fuzzy number to give a method for ranking type-2 fuzzy numbers; Liang and Mendel [3] proposed the concept of an interval type-2 fuzzy set for dealing with the operations via interval arithmetics; Zeng and Liu [4] described the important advances concerning type-2 fuzzy sets for pattern recognition, and in [5] explored the calculation of the union and intersection of concave type-2 fuzzy sets using the minimum t-norm and the maximum t-conorm. From the computational viewpoint, type-2 fuzziness is more difficult to deal with than type-1 fuzziness because the possibility of a type-2 fuzzy variable taking on a crisp value is a fuzzy number in [0, 1]. To avoid this difficulty, some type reduction approaches have been proposed in the literature for dealing with type-2 fuzziness, for example: [6] proposed a defuzzification method with the concept of a centroid of a type-2 fuzzy set; Liu [7] employed a centroid type reduction strategy for a general type-2 fuzzy logic system, and Qiu et al. [8] developed a statistical method for deciding on interval-valued fuzzy membership functions and a probability type reduction reasoning method for use with the interval-valued fuzzy logic system. In this paper, we attempt to present some novel reduction methods based on CVs of RFVs. According to the fuzzy integral [9], we first define three kinds of CVs for an RFV, which are referred to as the optimistic CV, the pessimistic CV and the CV. Some numerical examples are provided to illustrate the concepts, and the properties of CVs of trapezoidal, triangular, normal and gamma RFVs are also

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discussed. Furthermore, we develop three methods of reduction for type-2 fuzzy variables, which are called the optimistic, the pessimistic and the CV reduction methods, respectively.

In the literature, DEA technology was first proposed in [10]. One of the advantages of the DEA method is that it does not require either a priori weights for the inputs and outputs or the explicit specification of functional relations between the multiple inputs and outputs; therefore DEA has been widely used in many areas (see, e.g., [11–15]). A number of researchers have developed, in addition to the CCR model, some other meaningful DEA models, including the BCC model [16], the FDH (free disposal hull) model [17], the SBM (slack-based measure of efficiency) model [18], the RAM model [19] and so on. More advanced treatments may be found in [20,21]. On the basis of these models, some researchers extended the crisp inputs and outputs of traditional DEA models to stochastic data and developed some stochastic DEA models. For example, Sengupta [22] incorporated stochastic input and output variations into the DEA model; Banker [23] incorporated stochastic variables into DEA and developed a nonparametric approach; Cooper et al. [24] and Land et al. [25] developed a chance-constrained programming for DEA problems in order to accommodate the stochastic variations in the data. On other hand, fuzzy DEA with the inputs and outputs as fuzzy data has also been an area of active investigation. For instance, Sengupta [26] explored the use of fuzzy set-theoretic measures in the context of data envelopment analysis, and utilized a nonparametric approach for measuring efficiency; Triantis and Girod [27] suggested a mathematical programming approach to transforming fuzzy input and output data into crisp data by using membership function values; Wang and Yang [28], and Wang et al. [29] developed some methods for measuring the performance of DMUs, in which the efficiency is measured within the range of an interval; Wen and Li [30] established a DEA model in fuzzy environments and provided a ranking method for comparing the efficiencies of DMUs. On the basis of fuzzy possibility theory [31], this paper also considers data variations, and models fuzzy DEA from a new viewpoint, in which the inputs and outputs are characterized by type-2 fuzzy variables with known secondary possibility distributions. From the computational viewpoint, type-2 fuzziness is very complex compared with type-1 fuzziness. To overcome this difficulty, we first employ the proposed reduction methods in order to reduce the type-2 fuzzy inputs and outputs, then formulate a generalized credibility DEA model. When the inputs and outputs are mutually independent type-2 triangular fuzzy variables, we can turn the established DEA model into its equivalent parametric programming form, where the parameters can be used to characterize the degree of uncertainty as regards type-2 fuzziness. For any given parameters, the equivalent parametric programming model becomes a linear programming one that can be solved using standard optimization solvers. At the end of this paper, we provide one numerical example to illustrate the modeling idea and the efficiency in the proposed model by adjusting parameters with different values.

The rest of this paper is organized as follows. Section 2 introduces some concepts of type-2 fuzzy theory. In Section 3, we define three kinds of CVs for a fuzzy variable via the fuzzy integral and discuss the properties of CVs. In Section 4, we first develop the CV-based reduction methods for type-2 fuzzy variables, then discuss the fundamental properties for generalized credibility. In Section 5, we apply our reduction methods to the DEA model with type-2 fuzzy coefficients. Section 6 provides one numerical example to illustrate the modeling idea and the efficiency in the proposed DEA model. Section 7 gives our conclusions.

2. Fundamental concepts

Let Γ be the universe of discourse. An ample field [32] $\mathcal{A}$ on Γ is a class of subsets of Γ that is closed under arbitrary unions, intersections, and complements in Γ.

Let $\text{Pos} : \mathcal{A} \mapsto [0, 1]$ be a set function on the ample field $\mathcal{A}$. Pos is said to be a possibility measure [32] if it satisfies the following conditions:

(P1) $\text{Pos}(\emptyset) = 0$ and $\text{Pos}(\Gamma) = 1$.
(P2) For any subclass $\{A_i \mid i \in I\}$ of $\mathcal{A}$ (finite, countable or uncountable),

$$\text{Pos}\left(\bigcup_{i\in I} A_i\right) = \sup_{i\in I} \text{Pos}(A_i).$$

The triplet $(\Gamma, \mathcal{A}, \text{Pos})$ is referred to as a possibility space, in which a credibility measure [33] is defined as

$$\text{Cr}(A) = \frac{1}{2}(1 + \text{Pos}(A) - \text{Pos}(A^c)), \quad A \in \mathcal{A}.$$

If $(\Gamma, \mathcal{A}, \text{Pos})$ is a possibility space, then an m-ary regular fuzzy vector $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ is defined as a measurable map from Γ to the space $[0, 1]^m$ in the sense that for every $t = (t_1, t_2, \ldots, t_m) \in [0, 1]^m$, one has

$$\{\gamma \in \Gamma \mid \xi(\gamma) \le t\} = \{\gamma \in \Gamma \mid \xi_1(\gamma) \le t_1, \xi_2(\gamma) \le t_2, \ldots, \xi_m(\gamma) \le t_m\} \in \mathcal{A}.$$

When m=1, ξ is called a regular fuzzy variable (RFV).

In this paper, we denote by $\mathcal{R}([0, 1])$ the collection of all RFVs on [0, 1]. In the following, we provide several common RFVs.