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Resolution of computational aeroacoustics problems on unstructured grids with a higher-order finite volume scheme☆

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ABSTRACT

Computational fluid dynamics (CFD) has become increasingly used in the industry for the simulation of flows. Nevertheless, the complex configurations of real engineering problems make the application of very accurate methods that only work on structured grids difficult. From this point of view, the development of higher-order methods for unstructured grids is desirable. The finite volume method can be used with unstructured grids, but unfortunately it is difficult to achieve an order of accuracy higher than two, and the common approach is a simple extension of the one-dimensional case. The increase of the order of accuracy in finite volume methods on general unstructured grids has been limited due to the difficulty in the evaluation of field derivatives. This problem is overcome with the application of the Moving Least Squares (MLS) technique on a finite volume framework. In this work we present the application of this method (FV-MLS) to the solution of aeroacoustic problems.

1. Introduction

The simulation of sound propagation in the air is a very difficult numerical problem [1]. If we try to solve an acoustic problem with the same methods as developed for aerodynamics, a lot of numerical difficulties arise that are not present in the resolution of aerodynamic problems. The origin of such difficulties relies on the nature of the acoustic problem. The low magnitude of acoustic waves makes the use of low dissipation schemes mandatory, and it complicates even more the problem of the boundary conditions. Thus, the acceptable amplitude of reflections caused by waves leaving the domain is much smaller than in typical aerodynamic problems. Another feature of aeroacoustic problems is that the range of frequencies of interest is wider than in aerodynamics.

In computational aeroacoustics (CAA), the most successful numerical schemes have been spectral methods or high-resolution finite differences [2,3]. These methods work very well on structured grids, but unfortunately they present problems when applied to the resolution of problems with complex geometries. In this context, the development of methods that can solve CAA problems on unstructured grids is interesting. The finite volume method, widely and successfully used for the simulation of aerodynamics with unstructured grids, presents difficulties when it is applied to aeroacoustic problems in its most usual formulation (at most order two), due to the lack of resolution of the scheme. Even though raising the order

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is not the only (nor probably the best) way to improve the resolution of the schemes, it is the most usual approach on unstructured grids, due to the difficulty in generalizing the methods developed for structured meshes [4]. But this approach is also not obvious, and the main problem is the evaluation of high-order derivatives. The FV-MLS method [5–7] overcomes this difficulty by using the Moving Least Squares (MLS) technique [8] to compute the gradients and successive derivatives. Thus, it builds higher-order schemes in a finite volume framework without the introduction of new degrees of freedom.

The aim of this work is to extend the application of the FV-MLS method to the resolution of aeroacoustic problems, by focusing our attention on the resolution of the Linearized Euler Equations (LEEs). Moreover, the multiresolution features of the MLS approach [9] allow the development of low-pass filters that could be used together with a grid-stretching technique to build an absorbing layer that avoids reflections at the boundaries, following the methodology exposed in [10].

2. Linearized Euler equations

Most aeroacoustic problems are linear, so it is possible to linearize the Euler equations around a (mean) stationary solution $\boldsymbol{U_0} = (\rho_0, u_0, v_0, p_0)$. Then, the 2D LEEs written in conservative form are the following:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{F}}{\partial y} + \boldsymbol{H} = \boldsymbol{S} \tag{1}$$

where $\boldsymbol{S}$ is a source term and

$$\boldsymbol{U} = \begin{pmatrix} \rho' \\ u' \\ v' \\ p' \end{pmatrix} \qquad \boldsymbol{E} = \begin{pmatrix} \rho' u_0 + \rho_0 u' \\ \dfrac{p'}{\rho_0} + u_0 u' \\ u_0 v' \\ u_0 p' + \gamma p_0 u' \end{pmatrix} \qquad \boldsymbol{F} = \begin{pmatrix} \rho' v_0 + \rho_0 v' \\ v_0 u' \\ \dfrac{p'}{\rho_0} + v_0 v' \\ v_0 p' + \gamma p_0 v' \end{pmatrix} \tag{2}$$

$$\boldsymbol{H} = \begin{pmatrix} 0 \\ \dfrac{(\rho_0 u' + u_0 \rho')}{\rho_0} \dfrac{\partial u_0}{\partial x} + \dfrac{(\rho_0 v' + v_0 \rho')}{\rho_0} \dfrac{\partial u_0}{\partial y} \\ \dfrac{(\rho_0 u' + u_0 \rho')}{\rho_0} \dfrac{\partial v_0}{\partial x} + \dfrac{(\rho_0 v' + v_0 \rho')}{\rho_0} \dfrac{\partial v_0}{\partial y} \\ (\gamma - 1)\, p' \nabla \cdot \boldsymbol{v}_0 - (\gamma - 1)\, u' \nabla p_0 \end{pmatrix} \tag{3}$$

where the velocity is $\boldsymbol{v} = (u, v)$, ρ is the density, p the pressure, and $\gamma = 1.4$. The subscript $_0$ refers to mean values and $'$ indicates perturbation quantities around the mean. In case of a uniform mean flow, $\boldsymbol{H}$ is null.

3. Numerical method

3.1. An MLS-based finite volume scheme

A method based on the application of Moving Least Squares (MLS) to compute the derivatives in a finite volume framework (FV-MLS) [5,6] has been used to discretize the LEEs (1). Fluxes are discretized with a flux vector splitting method. In order to increase the order achieved by the method, a Taylor expansion of the variable is performed at the interior of each cell. Next, the approximation of the higher-order derivatives needed to compute the Taylor reconstruction is obtained by an MLS approach. Thus, if we consider a function $\Phi(\boldsymbol{x})$ defined in a domain Ω, the basic idea of the MLS approach is to approximate $\Phi(\boldsymbol{x})$, at a given point $\boldsymbol{x}$, through a weighted least-squares fitting of $\Phi(\boldsymbol{x})$ in a neighborhood of $\boldsymbol{x}$ as

$$\Phi(\boldsymbol{x}) \approx \widehat{\Phi}(\boldsymbol{x}) = \sum_{i=1}^{m} p_i(\boldsymbol{x})\, \boldsymbol{\alpha}_i(\boldsymbol{z})\, |_{\boldsymbol{z}=\boldsymbol{x}} = \boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x})\, \boldsymbol{\alpha}(\boldsymbol{z})\, |_{\boldsymbol{z}=\boldsymbol{x}}. \tag{4}$$

$\boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x})$ is an m-dimensional polynomial basis and $\boldsymbol{\alpha}(\boldsymbol{z})|_{\boldsymbol{z}=\boldsymbol{x}}$ is a set of parameters to be determined, such that they minimize the following error functional:

$$J(\boldsymbol{\alpha}(\boldsymbol{z})|_{\boldsymbol{z}=\boldsymbol{x}}) = \int_{\boldsymbol{y}\in\Omega_{\boldsymbol{x}}} W(\boldsymbol{z}-\boldsymbol{y}, h)|_{\boldsymbol{z}=\boldsymbol{x}} \left[\Phi(\boldsymbol{y}) - \boldsymbol{p}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{\alpha}(\boldsymbol{z})|_{\boldsymbol{z}=\boldsymbol{x}}\right]^2 \mathrm{d}\Omega_{\boldsymbol{x}}, \tag{5}$$

$W(\boldsymbol{z}-\boldsymbol{y}, h)|_{\boldsymbol{z}=\boldsymbol{x}}$ being a *kernel* with compact support (denoted by $\Omega_{\boldsymbol{x}}$) centered at $\boldsymbol{z} = \boldsymbol{x}$. The parameter h is the smoothing length, which is a measure of the size of the support $\Omega_{\boldsymbol{x}}$ [5].

In this work the following polynomial cubic basis is used:

$$\boldsymbol{p}(\boldsymbol{x}) = \begin{pmatrix} 1 & x & y & xy & x^2 & y^2 & x^2y & xy^2 & x^3 & y^3 \end{pmatrix}^{\mathrm{T}}, \tag{6}$$

which provides cubic completeness. In the above expression, (x, y) denotes the Cartesian coordinates of $\boldsymbol{x}$. In order to improve the conditioning, the polynomial basis is locally defined and scaled: if the shape functions are evaluated at $\boldsymbol{x}_I$,