Journal of Computational and Applied Mathematics 234 (2010) 2143–2151

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Dam break flow computation based on an efficient flux vector splitting

S. Erpicum [a], B.J. Dewals [a,b], P. Archambeau [a], M. Pirotton [a,*]

[a] HACH Research Unit, ArGEnCo Department, University of Liège, Chemin des Chevreuils, 1 B52/3, B-4000 Liège, Belgium

[b] Belgian National Fund for Scientific Research F.R.S-FNRS, Belgium

ARTICLE INFO

Article history:
Received 15 September 2008
Received in revised form 9 February 2009

Keywords:
Dam break flow
Shallow-water equations
Flux vector splitting

ABSTRACT

Dam break flow computation is a task of prime interest in the scope of risk analysis processes related to dams and reservoirs. In this paper, a 2D finite volume multiblock flow solver, able to deal with natural topography variation, is presented in detail. The model is based on an efficient Flux Vector Splitting method developed by the authors. A number of validation examples are comprehensively described.

1. Introduction

Dams and reservoirs are recognized for their valuable contribution to the prosperity and wealth of societies across the world, while they are also blamed for their risk of failure. Failures of large dams remain fortunately very seldom events. Nevertheless, a number of occurrences have been recorded in the world, corresponding in average to one to two failures worldwide every year. Some of those accidents have caused catastrophic consequences, but past experience reveals also that loss of life and damage can be drastically reduced if Emergency Action Planning (EAP) is implemented in the downstream valley. The development of EAP is an outcome of a complete risk analysis process, which requires, among other components, a detailed prediction of the propagation of the flood wave induced by the dam failure.

In this general scope, since practical risk analysis must be applicable for long real valleys at a sufficiently high space resolution, depth-averaged hydrodynamic models still constitute the most appealing approach [1]. Nevertheless, numerical modelling of such flows involves numerous challenges to be taken up. First, the flow is characterized by a highly transient behaviour involving the propagation of stiff fronts. Therefore, a proper upwind numerical scheme is required for the computation to be stable, while a satisfactory accuracy must be reached in space and time. Secondly, conservation of physical quantities must be preserved during wetting and drying of computation cells. Finally, the hydrodynamic model must be able to deal with flows over natural topographies, which are inherently irregular. The correct computation of momentum and energy quantities requires a suitable discretization of the source term representing the bottom slope.

Many authors such as Fread [2], Bellos and Sakkas [3], Glaister [4], Fennema and Chaudhry [5] or Alcrudo and al. [6] have shown early interest in dam break flow numerical computation. Various techniques have been developed to solve the 1D flow equations, as shown in [7]. In the early nineties, research was conducted at the University of Liege in [8], leading to a 1D upwind finite element scheme to compute dam break flows on natural topography and to its validation by physical modelling. The simulation time exceeded one week to run on 200 cells. Today, the corresponding 2D model is run in less than

* Corresponding address: ArGEnCo Department, Research unit of Applied Hydrodynamics and Hydraulic Constructions, University of Liege, Chemin des Chevreuils, 1 B52/3, B-4000Liege, Belgium.

E-mail addresses: S.Erpicum@ulg.ac.be (S. Erpicum), B.Dewals@ulg.ac.be (B.J. Dewals), Pierre.Archambeau@ulg.ac.be (P. Archambeau), Michel.Pirotton@ulg.ac.be (M. Pirotton).

0377-0427/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2009.08.110

one day on a 200,000-cell grid. Based on a finite volume scheme, this 2D hydrodynamic model handles multiblock regular grids and a specific iterative process to simulate wetting and drying of computation cells without mass or momentum error. Following extensive validation by comparison with theoretical, experimental and field data [9,10], this 2D model has proved its efficiency and reliability for open channel flow computations. In this paper, it is specifically applied to dam break flow computation.

2. Hydraulic model description

2.1. Mathematical model

The 2D hydraulic model is based on the 2D depth-averaged equations of volume and momentum conservation, namely the shallow-water equations (SWE). In the standard shallow-water approach, the only assumption states that vertical velocities are significantly smaller than horizontal ones. As a consequence, the pressure field is found to be almost hydrostatic everywhere. The large majority of flows occurring in natural valleys, even highly transient, can be reasonably seen as shallow, except in the vicinity of some singularities (e.g. weirs). Indeed, vertical velocity components remain generally low compared to velocity components in the horizontal plane and, consequently, flows may be considered as mainly 2D. Thus, the approach presented in this paper is suitable for many of the problems encountered in river management as well as for dam break modelling.

The conservative form of the depth-averaged equations of mass and momentum conservation can be written as follows, using vector notations:

$$\frac{\partial \mathbf{s}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} + \frac{\partial \mathbf{f}_d}{\partial x} + \frac{\partial \mathbf{g}_d}{\partial y} = \mathbf{S}_0 - \mathbf{S}_f \tag{1}$$

where $\mathbf{s} = [h\ hu\ hv]^{\mathrm{T}}$ is the vector of the conservative unknowns. Vectors $\mathbf{f}$ and $\mathbf{g}$ represent the advective and pressure fluxes in directions x and y, while $\mathbf{f}_d$ and $\mathbf{g}_d$ are the diffusive fluxes:

$$\begin{aligned} \mathbf{f} &= \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{pmatrix}, \qquad \mathbf{g} = \begin{pmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{pmatrix}, \\ \mathbf{f}_d &= -\frac{h}{\rho}\begin{pmatrix} 0 \\ \sigma_x \\ \tau_{xy} \end{pmatrix}, \qquad \mathbf{g}_d = -\frac{h}{\rho}\begin{pmatrix} 0 \\ \tau_{xy} \\ \sigma_y \end{pmatrix}. \end{aligned} \tag{2}$$

$\mathbf{S}_0$ and $\mathbf{S}_f$ designate respectively the bottom slope and the friction terms:

$$\mathbf{S}_0 = -gh\,[0\ \partial z_b/\partial x\ \partial z_b/\partial y]^{\mathrm{T}} \qquad \mathbf{S}_f = \left[0\ \tau_{bx}\Delta\Sigma/\rho\ \tau_{by}\Delta\Sigma/\rho\right]^{\mathrm{T}}. \tag{3}$$

In Eqs. (1)–(3), t represents the time, x and y the space coordinates, h the water depth, u and v the depth-averaged velocity components, z_b the bottom elevation, g the gravity acceleration, ρ the density of water, τ_{bx} and τ_{by} the bottom shear stresses, σ_x and σ_y the turbulent normal stresses and τ_{xy} the turbulent shear stresses. Consistently with Hervouet [11],

$$\Delta\Sigma = \sqrt{1 + (\partial z_b/\partial x)^2 + (\partial z_b/\partial y)^2} \tag{4}$$

reproduces the increased friction area on an irregular (natural) topography [12].

2.2. Friction modelling

The bottom friction is conventionally modelled thanks to an empirical law, such as the Manning formula. The model enables the definition of a spatially distributed roughness coefficient to represent different land-uses, floodplain vegetations or sub-grid bed forms . . . Besides, the model provides the additional possibility to reproduce friction along the side walls by means of a process-oriented formulation [9,12]. The combined effect of bottom and side wall friction is expressed as:

$$\begin{aligned} \frac{\tau_{bx}}{\rho} &= ghu\left[\sqrt{u^2+v^2}\,\frac{n_b^2}{h^{4/3}} + u\sum_{k_x=1}^{N_x}\frac{4}{3}\frac{n_W^2}{h^{1/3}\Delta y}\right] \\ \frac{\tau_{by}}{\rho} &= ghv\left[\sqrt{u^2+v^2}\,\frac{n_b^2}{h^{4/3}} + v\sum_{k_y=1}^{N_y}\frac{4}{3}\frac{n_W^2}{h^{1/3}\Delta x}\right] \end{aligned} \tag{5}$$

where the Manning coefficient n_b and n_w characterize respectively the bottom and the side walls roughness and N_x and N_y designate the number of edges of the finite volume cell which are in contact with the side wall. Those relations are particularized for Cartesian grids exploited in the present study.