An improved constitutive model for the numerical simulation of semi-solid thixoforming

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A B S T R A C T

In order to model thixoforming processes, previous papers presented a thermomechanical one-phase modelling. This first version of the constitutive model revealed several limitations: the model could not degenerate properly to pure solid or liquid behavior nor to free solid suspensions. The aim of this paper was to propose solutions to overcome these limitations.

1. Introduction

Semi-solid thixoforming is a forming process at temperatures located inside the fusion interval. It relies on a semi-solid microstructure (represented on the micrograph in Fig. 4 and illustrated in Fig. 2) made of globular solid grains more or less connected to each other, thus forming a solid skeleton deforming into a liquid phase. This particular microstructure makes semi-solid materials behave as solids at rest and flow during shearing because the viscosity and the resistance to deformation decrease with shearing, as illustrated in Fig. 1.

In a previous paper [1], we presented a thermomechanical one-phase modelling in order to predict die filling. In this kind of model, the material is regarded as a single continuous phase and the relative displacement between the phases cannot be taken into account. This first version of the constitutive model revealed several limitations: the model could not degenerate properly to pure solid or liquid behavior nor to isolated solid agglomerates. So several enhanced models have been proposed and implemented into the finite element code METAFOR.

2. Description of the proposed models

The basic idea is to extend the Norton–Hoff law to solid hypoelastic formulation, considering the elastic part of the deformation as well as two non-dimensional internal parameters: the **liquid fraction** and the **cohesion degree**.

Nowadays, there is still a dispute over whether thixotropic semi-solid alloys display yield or not. We decided here to use a finite yield stress since a vertical billet does not collapse under its own weight unless the liquid fraction is too high. Furthermore, this choice allows us to predict the residual stresses due to elasticity.

Semi-solid contains both liquid and solid and behaves as a solid or a fluid depending on process conditions. Behavior of liquids and solids under normal pressure are comparable, so we will focus on the deviatoric stresses to look for the main differences between those formalisms.

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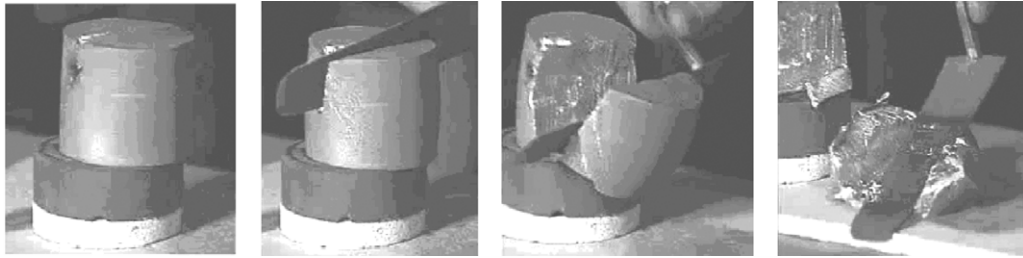*E-mail address:* r.koeune@ulg.ac.be (R. Koeune).

Fig. 1. Photographic sequence illustrating the thixotropic behavior of semi-solid alloy slugs [2].

Liquids resist very weakly to shear and tensile solicitations. Liquids cannot stop themselves shearing because they cannot develop enough restoring force to balance to such a stress (as solids do). That means that any shear stress applied on a liquid leads to steadily increasing strain, thus to a flow. Therefore, it does not make sense to **relate stress** to strain (as it is done for solids), but rather **to strain rate**. In the incompressible linear (Newtonian) case, we have:

$$\text{Liquid approach:} \quad \mathbf{s} = \eta \hat{\mathbf{D}} \tag{1}$$

where $\mathbf{s}$ and $\hat{\mathbf{D}}$ are the deviatoric stresses and strain rate tensor respectively and η is the viscosity.

On the other hand, as a **solid** is deformed beyond a certain level of strain, permanent, non-recoverable or plastic deformation occurs. Thus, the plastic behavior depends on the loading history and is mathematically represented by an **incremental stress–strain relationship**. In the linear (elastic) case, we have:

$$\text{Solid approach:} \quad \overset{\triangledown}{\mathbf{s}} = G \hat{\mathbf{D}} \tag{2}$$

where the operator $\triangledown$ stands for the objective time derivative and G is the shear modulus.

We can see that if the deformation stops ($\hat{\mathbf{D}} \rightarrow 0$ which does not mean that there is no strain left), the deviatoric stresses given by the fluid law (Eq. (1)) tends to zero for whatever viscosity. Therefore, the choice of a non-rigid solid formalism is motivated by the fact that such formalism offers the possibility to analyse the residual stresses after unloading and cooling down to room temperature.

Furthermore, if solids are submitted to higher stresses than their plasticity threshold (which decreases if temperature increases), a **plastic flow** is developed and some hardening occurs. At high temperatures both yield strength and strain hardening are really low and the solid material will deform by viscous flow. In this visco-plastic (non-elastic) case, an internal parameter describing the memory of the material is used. It is the equivalent plastic strain $\overline{\epsilon}^{vp}$, defined incrementally by mean of its rate: $\dot{\overline{\epsilon}}^{vp} = \sqrt{2/3\, \mathbf{D}^{vp} : \mathbf{D}^{vp}}$ which is calculated by an extended consistency equation:

$$\overline{\sigma}^{VM} - \sigma_y(\dot{\overline{\epsilon}}^{vp}, \overline{\epsilon}^{vp}) - \eta(\dot{\overline{\epsilon}}^{vp}, \overline{\epsilon}^{vp})\dot{\overline{\epsilon}}^{vp} = 0 \tag{3}$$

where $\overline{\sigma}^{VM} = \sqrt{3/2\, \mathbf{s} : \mathbf{s}}$ is the equivalent von Mises stress, σ_y the yield stress and η is the **apparent viscosity** defined by analogy with liquid formalism. So, the so-called "solid" formalism is not restricted to solid material (at temperature under solidus) and the aim of this work is to develop a constitutive law that is able to describe the thixotropic behavior in the whole range of temperatures occurring during the process. Thus, we are looking for a model that can predict the behavior of a built-up semi-solid material, or of free solid suspensions, or even the elastic behavior (to describe the cooling down to room temperature and thus the residual stresses).

The internal parameters of liquid fraction f_l and cohesion degree λ have been introduced to simulate the complex rheology of semi-solid materials, under both steady-state and transient conditions. For example, the peak of viscosity at start of a fast loading should be appropriately reproduced.

Thus, the new extended consistency equation is written as

$$\overline{\sigma}^{VM} - \sigma_y(f_l, \lambda, \dot{\overline{\epsilon}}^{vp}, \overline{\epsilon}^{vp}) - \eta(f_l, \lambda, \dot{\overline{\epsilon}}^{vp}, \overline{\epsilon}^{vp})\dot{\overline{\epsilon}}^{vp} = 0. \tag{4}$$

Different models based on this formulation have been proposed. Both internal parameters introduced in the initial work have been enhanced and the evolution of the yield stress and the apparent viscosity with the internal parameters have been described by several hardening and viscosity laws.

2.1. Initial model [1]

2.1.1. Cohesion degree

The first internal parameter is the **cohesion degree** λ and is illustrated in Fig. 2. During the process, the material structure changes with the strain history due to the agglomeration of the particles and the breaking of the grain bonds. So, λ is a