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Use of geometry in finite element thermal radiation combined with ray tracing

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ABSTRACT

In heat transfer for space applications, the exchanges of energy by radiation play a significant role. In this paper, we present a method which combines the geometrical definition of the model with a finite element mesh. The geometrical representation is advantageous for the radiative component of the thermal problem while the finite element mesh is more adapted to the conductive part. Our method naturally combines these two representations of the model. The geometrical primitives are decomposed into cells. The finite element mesh is then projected onto these cells. This results in a ray tracing acceleration technique. Moreover, the ray tracing can be performed on the exact geometry, which is necessary if specular reflectors are present in the model. We explain how the geometrical method can be used with a finite element formulation in order to solve thermal situation including conduction and radiation. We illustrate the method with the model of a satellite.

1. Introduction

When designing a spacecraft (a satellite, a manned vehicle or an automated probe), thermal issues must be carefully addressed. In the case of a spacecraft in vacuum, heat is transferred by conduction and radiation. To quantify the radiative interactions between the surfaces, an adimensional number, called view factor and denoted by F_{i-j}, is defined. This view factor is a function of the geometry; its correct computation requires to be performed on the exact geometry. This constraint is even more necessary if there are specular reflectors in the model. In order to take into account the different multi-reflections experienced by the radiation, common space thermal software uses stochastic ray tracing for the computation of the view factors, associated with finite difference approximation [1]. The drawback of this approach is the difficulty in the computation of the conductive component. This difficulty can easily be solved by using finite elements [2]. A solution developed in space thermal engineering consists of computing the conductive component, based on a finite element model of the spacecraft [3]; then a model reduction is used to transfer the resulting links to the system level where a ray tracing process can be used to compute the radiative links. A survey of techniques for coupling conduction and radiation can be found in [4].

In this paper, we present an original method which combines finite elements with the computation of the view factors with stochastic ray tracing based on the exact geometry. This method can be seen as the dual of the method developed in [3]. This method is also a ray tracing acceleration technique, allowing one to considerably reduce the CPU time. A survey of ray tracing acceleration methods can be found in [5]. The advantages of this method are detailed and numerical examples are given in this paper.

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2. Finite element discretization of the heat transfer equation

If we consider only steady-state problems, the temperature distribution through the volume $\mathcal{V}$, limited by the surface $\mathcal{S}$, is determined by the 3D heat transfer equation, where Q is the volume source [2]:

$$\mathrm{div}\left(k\,\overrightarrow{\mathrm{grad}T}\right)+Q=0\quad \text{on } \mathcal{V}. \tag{1}$$

The problem is completed with boundary conditions on temperature and heat flux:

$$\begin{cases} T(r)=f(r) & \text{on } \Gamma_T\\ q(r)=\aleph(r) & \text{on } \Gamma_q \end{cases} \tag{2}$$

where f and $\aleph$ are prescribed functions of the position r. The two domains must be complementary and cannot overlap:

$$\begin{cases} \Gamma_T\cap\Gamma_q=\emptyset\\ \Gamma_T\cup\Gamma_q=\mathcal{S}. \end{cases} \tag{3}$$

The normal heat flux q contains a radiative component q_r, which is a function of the fourth power of temperature [6]:

$$q_r(r)=\epsilon\sigma T^4(r)-\epsilon\left\{\int\!\!\int_{\mathcal{S}}\left[\sigma T^4(r')-\left(\frac{1}{\epsilon'}-1\right)q_r(r')\right]\mathrm{d}F_{r-r'}+H_0(r)\right\} \tag{4}$$

where ϵ is the local emissivity, σ is the Stefan–Boltzmann constant, equal to $5.67\ 10^{-8}\ \mathrm{W/m^2\ K^4}$; $dF_{r-r'}$ is the elementary view factor from point r to r'; $H_0(r)$ is the external irradiation incoming in r. The first term corresponds to the flux emitted by the surface while the second one represents the absorbed irradiation. This irradiation is the sum of two components: the first one is linked to the multi-reflections from the other surfaces of the model; the second one is due to the external irradiation $H_0(r)$.

The discretization of Eq. (1) with isoparametric finite elements yields a surface mesh where the boundary condition (4) is rewritten as follows:

$$\sigma T_i^4-\sum_{j=1}^{N}F_{i-j}\sigma T_j^4=\frac{q_{r,i}}{\epsilon_i}-\sum_{j=1}^{N}F_{i-j}\left(\frac{1}{\epsilon_j}-1\right)q_{r,j}+H_{0,i} \tag{5}$$

where the index i refers to surface elements and not to the finite element nodes. The temperature T_i is obtained by a linear combination of the nodal temperatures on i. The radiative flux $q_{r,i}$ is applied to the surface i and the view factors are computed from surface i to surface j. The view factors depend on the configuration of the surfaces *i.e.* on their sizes, their relative orientations and on the distance between them. By definition, the view factor between two surfaces A_i and A_j is the fraction of the uniform diffuse radiation leaving A_i that directly reaches A_j. The view factor is given by the Eq. (6):

$$F_{i-j}=\frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\cos(\theta_i)\cos(\theta_j)}{\pi s_{i-j}^2}V_{i-j}\mathrm{d}A_i\mathrm{d}A_j \tag{6}$$

where θ_i is the angle between the local normal $\overrightarrow{n_i}$ and the line connecting dA_i and dA_j; s_{i-j} is the distance between dA_i and dA_j. V_{i-j} is the visibility function. It is equal to 1 if the two points dA_i and dA_j can see each other. It is equal to 0 otherwise. This function introduces discontinuity in the view factor evolution.

In a limited set of simple geometrical configurations, this expression can be computed analytically. The evaluation of the view factor becomes more complex if there are obstacles in the 3D model. In this case, we need a numerical method and the geometrical model must be discretized. Among the available methods, a first class is based onto a deterministic integration scheme and is subject to aliasing. A second class is purely random, like the stochastic ray tracing. We select this class for its reliability and its flexibility.

3. Stochastic ray tracing

Among the stochastic methods, the most common one is the stochastic ray tracing. We generate a large number N_i of rays from the emitter A_i. Each ray is associated with an energy and can be considered as a bundle of photons. If the emitter is diffuse, the rays must obey the cosine law which governs the view factors. The direction of a ray is determined by two random numbers, denoted by ξ_1 and ξ_2, which are uniformly distributed between 0 and 1. We obtain:

$$\begin{cases} \theta=\arcsin\left(\sqrt{\xi_1}\right)\\ \phi=2\pi\xi_2 \end{cases} \tag{7}$$

where θ and ϕ correspond to the longitude and the latitude of the direction of the ray projected onto the unit hemisphere subtended by the origin [6].