Journal of Computational and Applied Mathematics 234 (2010) 950–959

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Linear bilevel programs with multiple objectives at the upper level ☆

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ARTICLE INFO

Article history:
Received 10 October 2008
Received in revised form 21 November 2008

MSC:
90C26
90C30

Keywords:
Bilevel programming
Multiobjective linear programming
Efficient
Non-dominated

ABSTRACT

Bilevel programming has been proposed for dealing with decision processes involving two decision makers with a hierarchical structure. They are characterized by the existence of two optimization problems in which the constraint region of the upper level problem is implicitly determined by the lower level optimization problem. Focus of the paper is on general bilevel optimization problems with multiple objectives at the upper level of decision making. When all objective functions are linear and constraints at both levels define polyhedra, it is proved that the set of efficient solutions is non-empty. Taking into account the properties of the feasible region of the bilevel problem, some methods of computing efficient solutions are given based on both weighted sum scalarization and scalarization techniques. All the methods result in solving linear bilevel problems with a single objective function at each level.

1. Introduction

General bilevel programming problems with a single objective function at each level can be formulated as:

$$\min_{x_1,x_2} \quad f_1(x_1,x_2), \tag{1a}$$

$$\text{s.t.} \quad (x_1,x_2) \in R \tag{1b}$$

where x_2 solves

$$\min_{x_2} \quad f_2(x_1,x_2) \tag{1c}$$

$$\text{s.t.} \quad (x_1,x_2) \in S \tag{1d}$$

where $x_1 \in \mathbb{R}^{n_1}$ are the upper level variables, which are controlled by the leader or upper level decision maker; $x_2 \in \mathbb{R}^{n_2}$ are the lower level variables, which are controlled by the follower or lower level decision maker; $f_1, f_2 : \mathbb{R}^n \longrightarrow \mathbb{R}$, $n = n_1 + n_2$, are the upper level and lower level objective functions, respectively; and $R, S \subseteq \mathbb{R}^n$ are the sets defined by the upper level and the lower level constraints, respectively.

These mathematical programs provide an appropriate model for hierarchical decision processes with two decision makers, the leader and the follower, each controlling part of the variables and having his own objective function and constraints. Bilevel problems have been increasingly addressed in the literature. Dempe [1] and Vicente and Calamai [2] provide surveys on the subject. Bard [3], Dempe [4] and Shimizu et al. [5] are good textbooks on this topic.

☆ This research work has been supported by the Spanish Ministry of Education and Science under grant MTM2007-66893.

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doi:10.1016/j.cam.2008.12.010

Optimization of a single objective function often oversimplifies processes involved in real systems. Most of these entail either a decision maker trying to get several goals or several decision makers each of them with his own objective. With regard to the last topic, Calvete and Galé [6] consider a linear bilevel problem with one leader and multiple followers which are assumed to be independent. In this problem all involved functions are linear and the objective function and the set of constraints of each follower only include the leader's variables and his own variables. They show that this problem can be reformulated into a linear bilevel problem with a single objective function at each level.

In order to deal with multiple and conflicting objectives, when there is a single level of decision making, several approaches have been proposed in the literature. Multiobjective programming refers to mathematical programs involving several objectives. This has been a very productive research field in mathematical programming during recent decades. Ehrgott [7], Ehrgott and Gandibleux [8] and Figueira et al. [9] provide a comprehensive overview of the literature.

Regarding the use of multiple objective functions in bilevel problems, Bonnel and Morgan [10] consider a semivectorial bilevel optimization problem in which the upper level is a scalar optimization problem, the lower level is a vector optimization problem and the constraints define Hausdorff topological spaces. They show that this bilevel problem with multiple objectives at the lower level can be approached using an exterior penalty method.

In this paper we are concerned by bilevel problems with multiple linear objective functions at the upper level and a linear objective function at the lower level. The constraint regions are assumed to be polyhedra. This model was motivated by a production–distribution planning problem in a supply chain. The distribution company, at the upper level, aims to minimize transportation cost as well as satisfy the preferences of retailers. Manufacturing plants, at the lower level, aim to minimize their own operation costs. First we reformulate the bilevel problem as a standard multiobjective problem with linear objective functions over a non-convex region. Next, we approach it from the multiobjective programming point of view, aiming to analyze its efficient set. We prove that the efficient set is non-empty and give some methods based on both weighted sum scalarization and scalarization techniques to obtain efficient points. Besides, several examples illustrate the complexity of the problem and show that some important properties held by linear multiobjective programs with a single level of decision making are no longer true, due to the lack of convexity. The paper is organized as follows. Section 2 states the problem. In Section 3 the existence of non-dominated points is proved. Sections 4 and 5 provide the main results on finding efficient and weakly efficient solutions. Finally, Section 6 concludes the paper with a summary.

2. Linear bilevel programs with multiple objective functions at the upper level

The linear multiobjective/linear bilevel programming (LMOLBP) problem can be formulated as:

$$\min_{x_1,x_2} \quad (d_1(x_1,x_2),\ldots,d_k(x_1,x_2)) \tag{2a}$$

$$\text{s.t.} \quad A_1^1x_1 + A_2^1x_2 \leqslant b^1 \tag{2b}$$

$$x_1 \geqslant 0 \tag{2c}$$

where x_2 solves

$$\min_{x_2} \quad c_2x_2 \tag{2d}$$

$$\text{s.t.} \quad A_1^2x_1 + A_2^2x_2 \leqslant b^2 \tag{2e}$$

$$x_2 \geqslant 0, \tag{2f}$$

where $d_i(x_1,x_2) = d_{i1}x_1 + d_{i2}x_2$; $d_{i1} : 1 \times n_1$; $d_{i2} : 1 \times n_2$, $i = 1,\ldots,k$; $c_2 : 1 \times n_2$; $A_1^1 : m_1 \times n_1$; $A_1^2 : m_2 \times n_1$; $A_2^1 : m_1 \times n_2$; $A_2^2 : m_2 \times n_2$; $b^1 : m_1 \times 1$; $b^2 : m_2 \times 1$. We assume that the polyhedron R defined by the upper level constraints (2b) and (2c) is non-empty and the polyhedron S defined by the lower level constraints (2e) and (2f) is non-empty and bounded. The polyhedron defined by all constraints is called the constraint region of the (LMOLBP) problem and will be denoted by T. We assume that T is non-empty.

For a given $\tilde{x}_1$, the follower solves the lower level linear programming problem:

$$LP(\tilde{x}_1): \quad \begin{aligned} \min_{x_2} \quad & c_2x_2 \\ \text{s.t.} \quad & A_2^2x_2 \leqslant b^2 - A_1^2\tilde{x}_1 \\ & x_2 \geqslant 0. \end{aligned} \tag{3}$$

Let $M(\tilde{x}_1)$ be the set of optimal solutions to (3):

$$M(\tilde{x}_1) = \left\{ \tilde{x}_2 \in \mathbb{R}^{n_2} : \tilde{x}_2 \in \operatorname*{argmin}_{x_2}\{c_2x_2 : A_2^2x_2 \leqslant b^2 - A_1^2\tilde{x}_1, x_2 \geqslant 0\} \right\}.$$

The feasible region of problem (2), called the inducible region, is implicitly defined as:

$$\text{IR} = \{(x_1,x_2) : (x_1,x_2) \in T,\ x_2 \in M(x_1)\}.$$