



A Petrov–Galerkin method for solving the generalized equal width (GEW) equation

Thoudam Roshan

Department of Mathematics, Manipur University, Canchipur – 795003, Imphal, India

ARTICLE INFO

Article history:

Received 13 May 2010

Received in revised form 3 September 2010

MSC:

65N30

65D07

76B25

Keywords:

Petrov–Galerkin

Product approximation

Quadratic *B*-spline

Solitary waves

Solitons

ABSTRACT

The generalized equal width (GEW) equation is solved numerically by the Petrov–Galerkin method using a linear hat function as the test function and a quadratic *B*-spline function as the trial function. Product approximation has been used in this method. A linear stability analysis of the scheme shows it to be conditionally stable. Test problems including the single soliton and the interaction of solitons are used to validate the suggested method, which is found to be accurate and efficient. Finally, the Maxwellian initial condition pulse is studied.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

The equal width (EW) equation of the form

$$u_t + \varepsilon uu_x - \delta u_{xxt} = 0 \quad (1)$$

where ε and δ are positive constants and the subscripts x and t denote space and time derivatives respectively is a model nonlinear partial differential equation used for the simulation of one-dimensional nonlinear waves propagating in dispersive media. Peregrine [1] was the first to derive the regularized long wave (RLW) equation for modeling the development of an undular bore. Later on, Benjamin et al. [2] proposed the use of the RLW equation as preferred alternative over the more classical Korteweg–de Vries (KdV) equation for modeling a large class of physical phenomena. The EW equation which is less well known and was proposed in [3] is a description alternative to the more usual KdV and RLW equations. The solutions of this equation are kinds of solitary waves known as solitons whose shapes are not affected by collision. The EW equation was solved numerically by various methods such as the Galerkin [4–6], least squares [7], and collocation methods [8,9] etc. Indeed, the EW equation is a special case of the generalized equal width (GEW) equation of the form

$$u_t + \varepsilon u^p u_x - \delta u_{xxt} = 0 \quad (2)$$

where p is a positive integer. It is the model equation governing the wave phenomena in which the long wave incorporates the competing effects of nonlinearity and dispersion. In fluid problems, u is related to the vertical displacement of the water surface, whereas in the plasma applications u is the negative of the electrostatic potential [10].

The GEW equation was studied in [10,11] by applying the collocation method with the use of quadratic and cubic *B*-spline functions. Comparatively little work has been done for this case. In this work, the Petrov–Galerkin method is developed for

E-mail address: roshandmc@gmail.com.

the GEW equation using a linear hat function as the test function and a quadratic B -spline as the trial function. Here the proposed method is shown to represent accurately the migration of single solitary waves for the cases $p = 2, 3$ and 4. The interaction of solitary waves and other properties are also studied.

2. The Petrov–Galerkin method (PGM)

For convenience, the GEW equation (2) is rewritten in the form

$$u_t + \frac{\varepsilon}{p+1} (u^{p+1})_x - \delta u_{xxt} = 0. \quad (3)$$

Periodic boundary conditions on the region $a \leq x \leq b$ are assumed in the form

$$u(a, t) = u(b, t) = 0. \quad (4)$$

The space interval $a \leq x \leq b$ is discretized with $(N+1)$ uniform grid points $x_j = a + jh$, where $j = 0, 1, 2, \dots, N$, and the grid spacing is given by $h = (b-a)/N$. Let $U_j(t)$ denote the approximation to the exact solution $u(x_j, t)$. Following the method used in [12] for solving the KdV equation, by the Petrov–Galerkin method we modify the method in order to solve Eq. (3). Using the Petrov–Galerkin method, we assume that the approximate solution of Eq. (3) is

$$u_h(x, t) = \sum_{j=0}^N U_j(t) \phi_j(x). \quad (5)$$

The product approximation technique [13] is used for treating the nonlinear term in the following manner:

$$u_h^{p+1}(x, t) = \sum_{j=0}^N U_j^{p+1}(t) \phi_j(x) \quad (6)$$

where $\phi_j(x)$, $j = 0, 1, 2, \dots, N$, are the usual piecewise linear “hat” functions given by

$$\phi_j(x) = \begin{cases} 1 + (x - jh)/h, & x \in [x_{j-1}, x_j] \\ 1 - (x - jh)/h, & x \in [x_j, x_{j+1}] \\ 0, & \text{otherwise.} \end{cases}$$

The unknown functions $U_j(t)$, $j = 0, 1, 2, \dots, N$, are determined from the variational formulation

$$\left((u_h)_t, \psi_j \right) + \frac{\varepsilon}{p+1} \left((u_h^{p+1})_x, \psi_j \right) - \delta \left((u_h)_{xxt}, \psi_j \right) = 0 \quad (7)$$

where ψ_j , $j = 0, 1, 2, \dots, N$, are test functions, which are taken to be quadratic B -splines given by

$$\psi_j(x) = \frac{1}{h^2} \begin{cases} (x - x_{j-1})^2, & x_{j-1} \leq x \leq x_j \\ 2h^2 - (x_{j+1} - x)^2 - (x - x_j)^2, & x_j \leq x \leq x_{j+1} \\ (x_{j+2} - x)^2, & x_{j+1} \leq x \leq x_{j+2} \\ 0, & \text{otherwise} \end{cases}$$

and $(\cdot, \cdot)$ denotes the usual inner product:

$$(f, g) = \int_a^b f(x)g(x)dx.$$

Integrating by parts and using the fact that $\psi(a) = \psi(b) = 0$, Eq. (7) leads to the formulation

$$\left((u_h)_t, \psi_j \right) + \frac{\varepsilon}{p+1} \left((u_h^{p+1})_x, \psi_j \right) + \delta \left((u_h)_{xt}, (\psi_j)_x \right) = 0. \quad (8)$$

Performing the integrations on (8) will give the following system of ordinary differential equations (ODEs):

$$\alpha \dot{U}_{j-1} + \beta \dot{U}_j + \beta \dot{U}_{j+1} + \alpha \dot{U}_{j+2} = \frac{2\varepsilon}{9(p+1)h} \left[(U_{j-1})^{p+1} + 3(U_j)^{p+1} - 3(U_{j+1})^{p+1} - (U_{j+2})^{p+1} \right] \quad (9)$$

where $\alpha = \frac{1}{18} \left(1 - \frac{12\delta}{h^2} \right)$, $\beta = \frac{1}{18} \left(11 + \frac{12\delta}{h^2} \right)$ and ‘ $\cdot$ ’ means the derivative with respect to time t .

Now to solve the ODEs, we assume U_j^n to be a fully discrete approximation to the exact solution $u(x_j, t_n)$, where $t_n = n\Delta t$ and Δt is the time step size. Using the central difference scheme for the time derivative, $\dot{U} = \frac{U^{n+1} - U^{n-1}}{2\Delta t}$, Eq. (9) is reduced to the system of equations

$$\begin{aligned} \alpha U_{j-1}^{n+1} + \beta U_j^{n+1} + \beta U_{j+1}^{n+1} + \alpha U_{j+2}^{n+1} &= \alpha U_{j-1}^{n-1} + \beta U_j^{n-1} + \beta U_{j+1}^{n-1} + \alpha U_{j+2}^{n-1} \\ &+ \frac{4\Delta t \varepsilon}{9(p+1)h} \left[(U_{j-1}^n)^{p+1} + 3(U_j^n)^{p+1} - 3(U_{j+1}^n)^{p+1} - (U_{j+2}^n)^{p+1} \right] \end{aligned} \quad (10)$$

where $j = 1, 2, 3, \dots, N-2$ and $n = 1, 2, 3, \dots$

Download English Version:

<https://daneshyari.com/en/article/4640523>

Download Persian Version:

<https://daneshyari.com/article/4640523>

[Daneshyari.com](https://daneshyari.com)