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Assessing the lifetime performance index of Rayleigh products based on the Bayesian estimation under progressive type II right censored samples

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ABSTRACT

Lifetime performance assessment is important in service (or manufacturing) industries. Hence, lifetime performance index C_L is used to measure the potential and performance of a process, where L is the lower specification limit. In this paper, assuming the conjugate prior distribution and squared-error loss function, this study constructs a Bayes estimator under the Rayleigh distribution with the progressive type II right censored sample. The Bayes estimator of C_L is then utilized to develop a credible interval in the condition of known L. Moreover, we also propose a Bayesian test to assess the lifetime performance of products. Finally, we give two examples and the Monte Carlo simulation to assess the behavior of the lifetime performance index C_L. Moreover, the purchasers can then employ the credible interval and the Bayesian test to determine whether the product performance adheres to the required level.

1. Introduction

Effective management and assessment of quality performance for products is important in modern enterprises. Process capability analysis is an effective means of measuring process performance and potential capability. In the service (or manufacturing) industry, process capability indices are utilized to assess whether product quality meets the required level. For instance, Montgomery [1] (or Kane [2]) proposed the process capability index C_L (or C_{PL}) for evaluating the lifetime performance of electronic components, where L is the lower specification limit, since the lifetime of electronic components exhibits the larger-the-better quality characteristic of time orientation. Tong et al. [3] constructed a uniformly minimum variance unbiased estimator (*UMVUE*) of C_L based on the complete sample from an exponential distribution. Moreover, the *UMVUE* of C_L is then utilized to develop the confidence interval. Chen et al. [4] also use the *UMVUE* of C_L to develop the confidence interval under an exponential distribution with the complete sample. The purchasers can then employ the testing procedure to determine whether the lifetime of electronic components adheres to the required level. Manufacturers can also utilize this procedure to enhance process capability.

All of the above PCIs have been developed or investigated under normal lifetime model or exponential lifetime model. Nevertheless, in many processes including manufacturing processes and service processes, the assumption of normality is common in process capability analysis, and is often not valid. Therefore, the lifetime model of many products generally may possess a non-normal distribution including exponential, Rayleigh or the other distributions and so forth. Since the Rayleigh distribution is also a special case of the Weibull distribution and has wide applications, such as, in the field of

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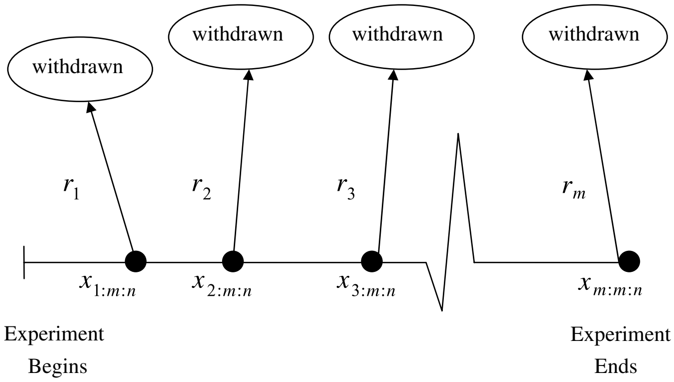

Fig. 1. Schematic representation of a progressive type II right censored sample.

acoustics (see [5,6]), in communication engineering (see [7,8]), in life testing of electrovacuum devices (see [9]), so in this paper we consider the lifetime of the product with Rayleigh distribution.

In life testing experiments, the experimenter may not always be in a position to observe the life times of all the products (or items) put on test. This may be because of time limitation and/or other restrictions (such as money, mechanical or experimental difficulties, material resources, etc.) on data collection. Therefore, censored samples may arise in practice. And, in an industrial experiment, products (or items) may break accidentally. These lead us into the area of progressive type II censoring. Inferences for the data that are obtained by progressive censoring have been investigated, among others, in [10–16]. So, in this paper, we consider the case of the progressive type II right censoring. A schematic illustration of a progressive type II right censored sample is depicted in Fig. 1, where $x_{1:m:n}, x_{2:m:n}, \ldots, x_{m:m:n}$ denote the observed failure times and $r_1, r_2, \ldots, r_m$ denote the corresponding number of items removed (withdrawn) from the test. Let m be the number of failures observed before termination and $x_{1:m:n} \le x_{2:m:n} \le \cdots \le x_{m:m:n}$ be the observed ordered lifetimes. Let r_i denote the number of items removed at the time of the ith failure, $0 \le r_i \le n - \sum_{j=1}^{i-1} r_j - i,\ i = 2, 3, \ldots, m-1$, with $0 \le r_1 \le n-1$ and $r_m = n - \sum_{j=1}^{m-1} r_j - m$, where r_i's and m are pre-specified integers (see [17]). The complete sample $(r_1 = r_2 = \cdots = r_m = 0)$ and type II right censored samples $(r_1 = r_2 = \cdots = r_{m-1} = 0, r_m = n - m)$ are special cases of this scheme. For further details and relevant references see [13]. Moreover, Hong et al. [18] constructed a maximum likelihood estimator (*MLE*) of C_L and developed a confidence interval for the performance index of businesses with the Pareto distribution based on the type II right censored samples. Lee et al. [19] proposed the lifetime performance index C_L based on the *MLE* with the exponential distribution under progressive type II right censored samples.

The Bayesian conception has received great attention for analyzing failure data and other time-to-event data. Bayes estimator and prediction problems have been discussed by several authors including Howlader and Hossain [20], Fernández [21], and AL-Hussaini and Ahmad [22,23]. In addition, Wu et al. [16] studied Bayes estimators and highest posterior density credible interval for the parameter and reliability function of Rayleigh distribution based on the progressive type II right censored samples.

The main aim of this paper is to construct a Bayes estimator of C_L under the Rayleigh distribution with the progressive type II right censored sample. The Bayes estimator of C_L is then utilized to develop a credible interval. The credible interval can be employed by managers to assess whether the product performance adheres to the required level in the condition of known L. In addition, a Bayesian test (also see [24, p. 379]) is also used to determine whether the product performance adheres to the required level.

The rest of this paper is organized as follows. Section 2 introduces some properties of the lifetime performance index for lifetime of product (or item) with the Rayleigh distribution based on the progressive type II right censored sample. Section 3 discusses the relationship between the lifetime performance index and the conforming rate. Section 4 then presents the Bayes estimator under the conjugate prior distribution and squared-error loss function of the lifetime performance index and its statistical properties. Moreover, we also propose the *MLE* of the lifetime performance index and its statistical properties. Section 5 develops a $100(1-\alpha)\%$ one-sided credible interval, a Bayesian test and a $100(1-\alpha)\%$ one-sided confidence interval for the lifetime performance index. Section 6 gives two numerical examples to illustrate the use of the testing procedure based on the Bayes estimator and the *MLE* under the given significance level. Section 7 discusses a Monte Carlo simulation of confidence level for $100(1-\alpha)\%$ one-sided credible interval and $100(1-\alpha)\%$ one-sided confidence interval of C_L based on the Bayes estimator $\hat{C}_L$ and the *MLE* $\tilde{C}_L$, respectively. Moreover, a comparison between the *MLE* and Bayes estimator is made through a Monte Carlo simulation study. Finally, concluding remarks are given in Section 8.

2. The lifetime performance index

Suppose that the lifetime (in years) of products (or items) may be modeled by a Rayleigh distribution. Let X years denote the lifetime of such a product (or item) and X has the Rayleigh distribution with the probability density function (p.d.f.) and