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An efficient computational method for linear fifth-order two-point boundary value problems

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ABSTRACT

In this paper, we present a new algorithm to solve general linear fifth-order boundary value problems (BVPs) in the reproducing kernel space $W_2^6[a, b]$. Representation of the exact solution is given in the reproducing kernel space. Its approximate solution is obtained by truncating the n-term of the exact solution. Some examples are displayed to demonstrate the computational efficiency of the method.

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1. Introduction

The solutions of the fifth-order BVPs are not often found in the literature. These problems generally arise in the mathematical modeling of viscoelastic flows and other branches of mathematical, physical and engineering sciences, see [1–4]. Agarwal's book [5] contains theorems which detail the conditions for existence and uniqueness of solutions of such BVPs. Recently, various powerful mathematical methods such as the sixth degree B-spline [6], Adomian decomposition methods [7], nonpolynomial sextic spline functions [8–12], local polynomial regression [13] and others [14,15] have been proposed to obtain exact and approximate analytic solutions for linear and nonlinear problems. Hikmet Caglar and Nazan Caglar [13] demonstrated the solution of fifth order boundary value problems by using local polynomial regression as follows:

$$\begin{cases} y^{(5)}(x) + f(x)y(x) = g(x), & a \leq x \leq b, \\ y(a) = \alpha_0, & y(b) = \alpha_1, \\ y'(a) = \gamma_0, & y'(b) = \gamma_1, \\ y''(a) = \delta_0. \end{cases} \tag{1.1}$$

Siddiqi, Akram and Malik [10] developed the nonpolynomial sextic spline method for special linear fifth-order two-point boundary value problems:

$$\begin{cases} y^{(5)}(x) + f(x)y(x) = g(x), & a \leq x \leq b, \\ y(a) = \alpha_0, & y(b) = \alpha_1, \\ y'(a) = \gamma_0, & y'(b) = \gamma_1, \\ y^{(3)}(a) = \delta_0. \end{cases} \tag{1.2}$$

In this paper, we consider the general linear fifth-order BVPs of the form:

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$$u^{(5)}(x)+\sum_{i=0}^{4}f_i(x)u^{(i)}(x)=g(x),\quad a\le x\le b \tag{1.3}$$

with the following boundary conditions:

$$u(a)=\alpha_0,\qquad u(b)=\alpha_1,\qquad u'(a)=\gamma_0,\qquad u'(b)=\gamma_1,\qquad u^{(3)}(a)=\delta_0 \tag{1.4}$$

as a first, third-order derivatives on both edges of the domain where α_i, γ_i $(i=0,1)$ and δ_0 all are real constants, $g(x)$ and $f_i(x)$ $(i=0,1,2,3,4)$ are all continuous functions on $[a,b]$.

We consider the boundary conditions corresponding to the first and third derivatives. Eq. (1.3) together with boundary value condition (1.4) have been considered in [10]. In this paper, we present a new algorithm to solve general fifth-order two-point boundary value problems in the reproducing kernel space. The advantage of the approach lies in the fact that the $u_n^{(k)}(x)$ are uniformly convergent to $u^{(k)}(x)$ $(k=0,1,\ldots,5)$. Examples are given to illustrate the efficiency and implementation of the method. Comparisons are made to confirm the reliability of the method.

After homogenization of the boundary conditions, Eq. (1.3) with (1.4) can be transformed into the following form

$$\tilde{u}^{(5)}(x)+\sum_{i=0}^{4}\tilde{f}_i(x)\tilde{u}_i(x)=\tilde{g}(x),\quad a\le x\le b \tag{1.5}$$

subject to

$$\tilde{u}(a)=0,\qquad \tilde{u}(b)=0,\qquad \tilde{u}'(a)=0,\qquad \tilde{u}'(b)=0,\qquad \tilde{u}^{(3)}(a)=0. \tag{1.6}$$

For convenient, let $u(x)$ denote $\tilde{u}(x)$ and $g(x)$ denote $\tilde{g}(x)$. Taking the differential operator

$$\mathcal{L}u(x)=u^{(5)}(x)+\sum_{i=0}^{4}f_i(x)u_i(x), \tag{1.7}$$

we convert the above Eq. (1.5) with (1.6) as follows:

$$\begin{cases}\mathcal{L}u(x)=g(x),\quad a\le x\le b,\\ u(a)=0,\qquad u(b)=0,\\ u'(a)=0,\qquad u'(b)=0,\\ u^{(3)}(a)=0.\end{cases} \tag{1.8}$$

2. Several reproducing kernel spaces

In this section we introduce two reproducing kernel spaces.

2.1. The reproducing kernel space $W_2^6[a,b]$

$$\begin{aligned}W_2^6[a,b]=\{u(x)\,|\,&u^{(5)}(x) \text{ is an absolutely continuous real value function in } [a,b],\\ &u^{(6)}(x)\in L^2[a,b],\ u(a)=u(b)=u'(a)=u'(b)=u^{(3)}(a)=0\}\end{aligned} \tag{2.1}$$

and endowed it with the inner product and norm respectively

$$\langle u(x),v(x)\rangle_{W_2^6}=\int_a^b\left(u^{(5)}(x)v^{(5)}(x)+u^{(6)}(x)v^{(6)}(x)\right)\mathrm{d}x, \tag{2.2}$$

$$\|u\|_{W_2^6[a,b]}=\langle u,u\rangle^{\frac{1}{2}}. \tag{2.3}$$

Theorem 2.1. *The space $W_2^6[a,b]$ is a reproducing kernel space. That is, there exists a function $R_x(y)$, for each fixed $x\in[a,b]$, $R_x(y)\in W_2^6[a,b]$, and for any $u(y)\in W_2^6[a,b]$, satisfying*

$$\langle u(y),R_x(y)\rangle_{W_2^6}=u(x), \tag{2.4}$$

the reproducing kernel $R_x(y)$ can be denoted by

$$R_x(y)=\begin{cases}\displaystyle\sum_{i=1}^{10}a_i(x)y^{i-1}+a_{11}(x)\mathrm{e}^{y}+a_{12}(x)\mathrm{e}^{-y},\quad y\le x,\\ \displaystyle\sum_{i=1}^{10}b_i(x)y^{i-1}+b_{11}(x)\mathrm{e}^{y}+b_{12}(x)\mathrm{e}^{-y},\quad y>x,\end{cases} \tag{2.5}$$

where $a_i(x)$, $b_i(x)$ $(i=1,2,\ldots,10)$ are known coefficients.

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