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Stability analysis of stochastic functional differential equations with infinite delay and its application to recurrent neural networks ☆

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ABSTRACT

In this paper, we investigate the stochastic functional differential equations with infinite delay. Some sufficient conditions are derived to ensure the pth moment exponential stability and pth moment global asymptotic stability of stochastic functional differential equations with infinite delay by using Razumikhin method and Lyapunov functions. Based on the obtained results, we further study the pth moment exponential stability of stochastic recurrent neural networks with unbounded distributed delays. The result extends and improves the earlier publications. Two examples are given to illustrate the applicability of the obtained results.

1. Introduction

In recent years, there is an increasing interest in stochastic functional differential equations due to their important applications in practice [1–4], and a large number of interesting results of these equations have been reported; see [5–17]. For instance, in 2002, Taniguchi et al. [6] considered the existence, uniqueness, pth moment and almost sure Lyapunov exponents of mild solutions to a class of stochastic functional differential equations with finite delays by using semigroup methods. In 2006, Shen et al. [7] discussed the existence and uniqueness of solutions of stochastic functional differential equations with finite delay by using Lyapunov functions and quasi-local Lipschitz condition. On the other hand, it is well known that the method of Razumikhin technique and Lyapunov functions (or Lyapunov functional) have been very powerful and effective in the study of stability analysis of various delay differential equations; see [18–23,12,13]. In 1984, Chang [12] firstly established some Razumikhin-type uniformly asymptotic stability criteria of stochastic functional differential equations with finite delay. In 1996, Mao [13] further developed the Razumikhin method on this aspect and established some Razumikhin-type theorems on exponential stability of stochastic functional differential equations with finite delay. However, in the past ten years, little work has been done on this aspect due to theoretical and technical difficulties.

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On the other hand, although stochastic functional differential equations with infinite delay also provide many mathematical models for various phenomena and processes in the field of control engineering, electrical and physical sciences as well as stochastic functional differential equations with finite delay, the corresponding properties of these systems have not been developed in great detail; see [14–17]. Results on existence and uniqueness of the solutions of retarded stochastic functional differential equations with infinite delay have been presented in [16,17] via different approaches. As we know, some practical applications of stochastic functional differential equations with infinite delay are greatly dependent on the stability properties of their solutions. However, to the best of the authors' knowledge, so far there is almost no result of Razumikhin type on the stability of stochastic functional differential equations with infinite delay.

Motivated by the above discussions, in this paper we shall extend Razumikhin method to stochastic functional differential equations with infinite delay and establish some theorems on pth moment exponential stability and pth moment global asymptotic stability. As an application, we study the stochastic recurrent neural networks with unbounded distributed delays via the obtained result. Some conditions are obtained to ensure the pth moment exponential stability of stochastic recurrent neural networks with unbounded distributed delays. The obtained results are more general than those given in [24–27]. Finally, two illustrative examples are provided to show the effectiveness of our results.

2. Preliminaries

Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $|\cdot|$, and $\mathbb{Z}_+$ the set of positive integral numbers. $E(\cdot)$ stands for the mathematical expectation operator. $\mathscr{L}$ denotes the well-known $\mathscr{L}$-operator given by the Itô formula. $C([\alpha,0],\mathbb{R}^n)$ denotes the family of all continuous functions ϕ from $[\alpha,0]$ into $\mathbb{R}^n$. Let $\alpha\wedge\beta$ denote the minimum value of α and β. $\omega(t)=(\omega_1(t),\ldots,\omega_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathscr{F},P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$ generated by $\{\omega(s):0\leq s\leq t\}$, where we associate Ω with the canonical space generated by $\omega(t)$, and denoted by $\mathscr{F}$ the associated σ-algebra generated by $\omega(t)$ with the probability measure P.

In this paper, we consider the following n-dimensional stochastic functional differential equations with infinite delay:

$$\begin{cases} \mathrm{d}x(t)=f(t,x(t),x_t)\mathrm{d}t+\sigma(t,x(t),x_t)\mathrm{d}\omega(t), & t\geq 0,\\ x_0=\xi, \end{cases} \tag{1}$$

where the initial condition $\xi\in BC_{\mathscr{F}_0}([\alpha,0],\mathbb{R}^n)$, $x(t)=(x_1(t),\ldots,x_n(t))^T$ and $x_t=\{x(t+\theta):\alpha\leq\theta\leq 0\}$ which can be regarded as a $C([\alpha,0],\mathbb{R}^n)$-valued stochastic process, where $\alpha\in[-\infty,0)$. Especially when $\alpha=-\infty$, the interval $[t+\alpha,t]$ is understood to be replaced by $(-\infty,t]$. $f:\mathbb{R}_+\times\mathbb{R}^n\times C([\alpha,0],\mathbb{R}^n)\to\mathbb{R}^n$, $\sigma:\mathbb{R}_+\times\mathbb{R}^n\times C([\alpha,0],\mathbb{R}^n)\to\mathbb{R}^{n\times m}$. Denoted by $BC_{\mathscr{F}_0}([\alpha,0],\mathbb{R}^n)$ the family of all $\mathscr{F}_0$-measurable, $BC([\alpha,0],\mathbb{R}^n)$-valued random variables, where $BC([\alpha,0],\mathbb{R}^n)$ denote the family of all bounded $C([\alpha,0],\mathbb{R}^n)$-valued functions φ with the norm $\|\varphi\|_\alpha=\sup_{\alpha\leq\theta\leq 0}|\varphi(\theta)|$. For $p>0$ and $t\geq t_0$, denoted by $BL^p_{\mathscr{F}_t}([\alpha,0],\mathbb{R}^n)$ the family of all $\mathscr{F}_t$-measurable $BC([\alpha,0],\mathbb{R}^n)$-valued random variables φ, satisfying $\sup_{\alpha\leq\theta\leq 0}E|\varphi(\theta)|^p<\infty$. Especially, let $BC_{\mathscr{F}_0}\doteq BC_{\mathscr{F}_0}([\alpha,0],\mathbb{R}^n)$ and $BL^p_{\mathscr{F}_t}\doteq BL^p_{\mathscr{F}_t}([\alpha,0],\mathbb{R}^n)$.

As usual, throughout this paper, we assume that system (1) has a unique solution on $t\geq 0\geq\alpha$, which is denoted by $x(t,\xi)$ (see [16,17]). Moreover, we assume that $f(t,0,0)=0$, $\sigma(t,0,0)=0$ for the stability purpose of this paper. So system (1) admits an equilibrium solution $x(t)\equiv 0$.

Let $C^{2,1}([\alpha,\infty)\times\mathbb{R}^n\to\mathbb{R}_+)$ denote the family of all non-negative functions $V(t,x)$ on $[\alpha,\infty)\times\mathbb{R}^n$ which are continuous once differentiable in t and twice differentiable in x. For each such V, we define an operator $\mathscr{L}V$ associated with (1) as

$$\mathscr{L}V(t,\varphi)=V_t(t,\varphi(0))+V_x(t,\varphi(0))f(t,\varphi(0),\varphi)+\frac{1}{2}\mathrm{trace}[\sigma^T V_{xx}(t,\varphi(0))\sigma],$$

where

$$V_t(t,x)=\frac{\partial V(t,x)}{\partial t},\qquad V_x(t,x)=\left(\frac{\partial V(t,x)}{\partial x_1},\ldots,\frac{\partial V(t,x)}{\partial x_n}\right),\qquad V_{xx}(t,x)=\left(\frac{\partial^2 V(t,x)}{\partial x_i x_j}\right)_{n\times n}.$$

Definition 2.1 (*Mao [2], Luo [15]*). The equilibrium solution of system (1) is said to be

(P_1) pth moment globally asymptotically stable if for any $\xi\in BC_{\mathscr{F}_0}$,

$$E|x(t,\xi)|^p\to 0\quad \text{as } t\to\infty.$$

Especially when $p=2$, it is usually called to be globally asymptotically stable in mean square.

(P_2) pth moment exponentially stable if there exists a pair of positive constants $\mathbb{M}$ and λ such that for any $\xi\in BC_{\mathscr{F}_0}$,

$$E|x(t,\xi)|^p<\mathbb{M}E\|\xi\|^p_\alpha \mathrm{e}^{-\lambda t},\quad t\geq 0.$$

Especially when $p=2$, it is usually called to be exponentially stable in mean square.