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Optimality of the barrier strategy in de Finetti's dividend problem for spectrally negative Lévy processes: An alternative approach

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ABSTRACT

The optimal dividend problem proposed in de Finetti [1] is to find the dividend-payment strategy that maximizes the expected discounted value of dividends which are paid to the shareholders until the company is ruined. Avram et al. [9] studied the case when the risk process is modelled by a general spectrally negative Lévy process and Loeffen [10] gave sufficient conditions under which the optimal strategy is of the barrier type. Recently Kyprianou et al. [11] strengthened the result of Loeffen [10] which established a larger class of Lévy processes for which the barrier strategy is optimal among all admissible ones. In this paper we use an analytical argument to re-investigate the optimality of barrier dividend strategies considered in the three recent papers.

1. Introduction

This paper considers the classical optimal dividend control problem for a company. The idea is that the company wants to pay some its surplus to the shareholders as dividends, the problem is to find a dividend-payment strategy that maximizes the expected discounted value of all payments until the company's capital is negative for the first time. This optimization problem goes back to [1], who considered a discrete time random walk with step sizes ± 1 and proved that the optimal dividend strategy is a barrier strategy. Optimal dividend problem has recently gained a lot of attention in actuarial mathematics. It has been studied extensively in the diffusion process setting, see [2–5]. It is well known that under some reasonable assumptions, the optimality in the diffusion process setting is achieved by using a barrier strategy (see [4,5]). However, in the Cramér–Lundberg setting this is not the case; it was shown in [6] that the optimal dividend strategy is of so-called band type. This results was re-derived by means of viscosity theory in [7]. In particular, for exponentially distributed claim sizes this optimal strategy simplifies to a barrier strategy. The summary of Finetti and Gerber's work can be found in [8]. Recently, Avram et al. [9] considered the case where the risk process is given by a general spectrally negative Lévy process and gave a sufficient condition involving the generator of the Lévy process for the optimal strategy to consist of a barrier strategy. In [10], Loeffen defined an optimal barrier level which is slightly different than the one given in [9] and proved the remarkable fact that, if the q-scale function $W^{(q)}$ is convex in the interval (a^*, ∞), where

$$a^* = \sup\{a \geq 0 : W^{(q)\prime}(a) \leq W^{(q)\prime}(y) \text{ for all } y \geq 0\} < \infty,$$

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then the barrier strategy at a^* is an optimal strategy among all admissible strategies. Moreover, it is shown that when the Lévy measure Π of X has a completely monotone density, then $W^{(q)\prime}$ is strictly convex on $(0, \infty)$ for all $q > 0$. Consequently, the barrier strategy at a^* is an optimal strategy. In a very recent work [11], the authors prove a more general result: Suppose that the Lévy measure Π of X has a non-increasing density which is logconvex, then for $q > 0$ the scale function $W^{(q)}$ is convex in the interval (a^*, ∞). As a consequence, the barrier strategy at a^* is an optimal strategy. In the other recent paper, Albrecher and Thonhausera [12] discussed the maximization problem in a generalized setting including a constant force of interest in the Cramér–Lundberg risk model. The value function is identified in the set of viscosity solutions of the associated Hamilton–Jacobi–Bellman equation and the optimal dividend strategy in this risk model is derived, which in the general case is again of band type and for exponential claim sizes collapses to a barrier strategy.

In this paper, it is assumed that the surplus process is a general spectrally negative Lévy process, we provide an analytical study of the solution to the classical dividend control problem due to [9,11,10].

The rest of the paper is organized as follows. In Section 2, we recall some preliminaries on the spectrally negative Lévy process and state the problem. In Section 3, we will review some basic results on the logconvexity and complete monotonicity of the functions that will be needed later on. In Section 4 we discuss the convex solutions for two kinds of integro-differential equations and in Section 5 we present the main results and prove them by using the results of Section 4 and some earlier results from [9,11,10]. Finally, some remarks are included in Section 6.

2. The model

Suppose that $X = (X(t) : t \geq 0)$ is a spectrally negative Lévy process with probabilities $\{P_x : x \in \mathbb{R}\}$ such that $X(0) = x$ with probability one, where we write $P = P_0$. Let E_x be the expectation with respect to P_x and write $E = E_0$. That is to say X is a real valued stochastic process whose paths are almost surely right continuous with left limits and whose increments are stationary and independent. Let $\{\mathcal{F}_t : t \geq 0\}$ be the natural filtration satisfying the usual assumptions. Since the jumps of a spectrally negative Lévy process are all non-positive, the moment generating function $E(e^{\theta X(t)})$ exists for all $\theta \geq 0$ and is given by $E(e^{\theta X(t)}) = e^{t\psi(\theta)}$ for some function $\psi(\theta)$, which is called the Laplace exponent of X. From the Lévy–Khintchin formula [13,14], it is known that

$$\psi(\theta) = a\theta + \frac{1}{2}\sigma^2\theta^2 - \int_0^\infty \left(1 - e^{-\theta x} - \theta x \mathbf{1}_{\{0<x<1\}}\right) \Pi(dx) \tag{2.1}$$

where $a \in \mathbb{R}$, $\sigma \geq 0$ and Π is a measure on $(0, \infty)$ satisfying $\int_0^\infty (1 \wedge x^2)\Pi(dx) < \infty$ and is called the Lévy measure. ψ is strictly convex on $(0, \infty)$ and satisfies $\psi(0+) = 0$, $\psi(\infty) = \infty$ and $\psi'(0+) = EX(1)$. Further, ψ is strictly increasing on $[\phi(0), \infty)$, where $\phi(0)$ is the largest root of $\psi(\theta) = 0$ (there are at most two). We shall denote the right-inverse function of ψ by ϕ: $[0, \infty) \to [\phi(0), \infty)$. If $\sigma^2 > 0$ and $\Pi = 0$, then the process is a Brownian motion; When $\sigma^2 = 0$ and $\int_0^\infty \Pi(dx) < \infty$, the process is a compound Poisson process; when $\sigma^2 = 0$, $\int_0^\infty \Pi(dx) = \infty$ and $\int_0^\infty (1 \wedge x)\Pi(dx) < \infty$, the process has an infinite number of small jumps, but is of finite variation; when $\sigma^2 = 0$, $\int_0^\infty \Pi(dx) = \infty$ and $\int_0^\infty (1 \wedge x)\Pi(dx) = \infty$, the process has infinitely many jumps and is of unbounded variation. In short, such a Lévy process has bounded variation if and only if $\sigma = 0$ and $\int_0^1 x\Pi(dx) < \infty$. In this case the Lévy exponent can be re-expressed as

$$\psi(\alpha) = b\alpha - \int_0^\infty (1 - e^{\alpha x})\Pi(dx),$$

where $b = a - \int_0^1 x\Pi(dx)$ is known as the drift coefficient. If $\sigma^2 > 0$, X is said to have a Gaussian component.

For θ such that $\psi(\theta)$ is finite we denote by P_x^θ an exponential tilting of the measure P_x with a Radom–Nikodym derivative with respect to P_x given by

$$\left.\frac{dP_x^\theta}{dP_x}\right|_{\mathcal{F}_t} = \exp(\theta(X(t) - x) - \psi(\theta)t).$$

Under the measure P_x^θ the process X is still a spectrally negative Lévy process with Laplace exponent ψ_θ given by

$$\psi_\theta(\eta) = \psi(\eta + \theta) - \psi(\theta), \quad \eta \geq -\theta.$$

We recall from [15,13], that for each $q \geq 0$ there exits a continuous and increasing function $W^{(q)} : \mathbb{R} \to [0, \infty)$, called the q-scale function, defined in such a way that $W^{(q)}(x) = 0$ for all $x < 0$, and on $[0, \infty)$ its Laplace transform is given by

$$\int_0^\infty e^{-\theta x} W^{(q)}(x)dx = \frac{1}{\psi(\theta) - q}, \quad \theta > \phi(q). \tag{2.2}$$

For convenience we shall write W in place of $W^{(0)}$ and call this the scale function rather than the 0-scale function.

The following facts about the smoothness of the scale functions are taken from [11]. If X has paths of bounded variation then, for all $q \geq 0$, $W^{(q)}|_{(0,\infty)} \in C^1(0, \infty)$ if and only if Π has no atoms. In the case that X has paths of unbounded variation,