Journal of Computational and Applied Mathematics 234 (2010) 825–834

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Evaluation of fuzzy regression models by fuzzy neural network

M. Mosleh [a,1], M. Otadi [a,*], S. Abbasbandy [b,2]

[a] Department of Mathematics, Islamic Azad University, Firuozkooh Branch, Firuozkooh, Iran

[b] Department of Mathematics, Islamic Azad University, Science and Research Branch, Tehran 14515/775, Iran

ARTICLE INFO

Article history:
Received 4 July 2009
Received in revised form 22 January 2010

Keywords:
Fuzzy neural networks
Fuzzy regression model
Fuzzy number
Feedforward neural network
Learning algorithm

ABSTRACT

In this paper, a novel hybrid method based on fuzzy neural network for approximate fuzzy coefficients (parameters) of fuzzy linear and nonlinear regression models with fuzzy output and crisp inputs, is presented. Here a neural network is considered as a part of a large field called neural computing or soft computing. Moreover, in order to find the approximate parameters, a simple algorithm from the cost function of the fuzzy neural network is proposed. Finally, we illustrate our approach by some numerical examples.

1. Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced in [1,2]. We refer the reader to [3] for more information on fuzzy numbers and fuzzy arithmetic.

Regression analysis is one of the most popular methods of estimation. It is applied to evaluate the functional relationship between the dependent and independent variables. Fuzzy regression analysis is an extension of the classical regression analysis in which some elements of the model are represented by fuzzy numbers. Fuzzy regression methods have been successfully applied to various problems such as forecasting [4–6] and engineering [7]. Thus, it is very important to develop numerical procedures that can appropriately treat fuzzy regression models. Modarres et al. [8] proposed a mathematical programming model to estimate the parameters of a fuzzy linear regression

$$Y_i = A_1 x_{i1} + A_2 x_{i2} + A_n x_{in},$$

where $x_{ij} \in \mathbb{R}$ and $A_1, A_2, \ldots, A_n, Y_i$ are symmetric fuzzy numbers for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Ishibuchi et al. [9] proposed a learning algorithm of fuzzy neural networks with triangular fuzzy weights and Hayashi et al. [10] fuzzified the delta rule. Buckley and Eslami [11] consider neural network solutions to fuzzy problems. The topic of numerical solution of fuzzy polynomials by fuzzy neural network investigated in [12], consists of finding solution to polynomials like $a_1 x + a_2 x^2 + \cdots + a_n x^n = a_0$ where $x \in \mathbb{R}$ and $a_0, a_1, \ldots, a_n$ are fuzzy numbers, and finding solution to systems of s fuzzy polynomial equations such as [13]:

* Corresponding author. Tel.: +98 912 6964202.
E-mail addresses: Maryam_Mosleh79@yahoo.com (M. Mosleh), mahmood_otadi@yahoo.com (M. Otadi), abbasbandy@yahoo.com (S. Abbasbandy).
1 Tel.: +98 912 6076308.
2 Tel.: +98 912 1305326; fax: +98 281 3780040.

0377-0427/$ – see front matter
doi:10.1016/j.cam.2010.01.046

$$f_1(x_1, x_2, \dots, x_n) = a_{10},$$
$$\vdots$$
$$f_l(x_1, x_2, \dots, x_n) = a_{l0},$$
$$\vdots$$
$$f_s(x_1, x_2, \dots, x_n) = a_{s0},$$

where $x_1, x_2, \dots, x_n \in \mathbb{R}$ and all coefficients are fuzzy numbers.

In this paper, we first propose an architecture of fuzzy neural network (FNN) with fuzzy weights for real input vectors and fuzzy targets to find approximate coefficients to fuzzy linear regression model

$$\overline{Y}_i = A_0 + A_1 x_{i1} + \cdots + A_n x_{in},$$

and fuzzy nonlinear regression model

$$\overline{Y}_i = A_0 e^{A_1 x_{i1} + A_2 x_{i2} + A_n x_{in}},$$

where i indexes the different observations, $x_{i1}, x_{i2}, \dots, x_{in} \in \mathbb{R}$, all coefficients and $\overline{Y}_i$ are fuzzy numbers. The input–output relation of each unit is defined by the extension principle of Zadeh [1]. Output from the fuzzy neural network, which is also a fuzzy number, is numerically calculated by interval arithmetic [14] for fuzzy weights and real inputs. Next, we define a cost function for the level sets of fuzzy outputs and fuzzy targets. Then, a crisp learning algorithm is derived from the cost function to find the fuzzy coefficients of the fuzzy linear and nonlinear regression models. The proposed algorithm is illustrated by some examples in the last section.

2. Preliminaries

In this section the basic notations used in fuzzy calculus are introduced. We start by defining the fuzzy number.

Definition 1. A fuzzy number is a fuzzy set $u : \mathbb{R}^1 \longrightarrow I = [0, 1]$ such that

i. u is upper semi-continuous;
ii. $u(x) = 0$ outside some interval $[a, d]$;
iii. There are real numbers b and c, $a \le b \le c \le d$, for which
   1. $u(x)$ is monotonically increasing on $[a, b]$,
   2. $u(x)$ is monotonically decreasing on $[c, d]$,
   3. $u(x) = 1, b \le x \le c$.

The set of all the fuzzy numbers (as given in Definition 1) is denoted by E^1.

An alternative definition which yields the same E^1 is given in [15,16].

Definition 2. A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \le r \le 1$, which satisfy the following requirements:

i. $\underline{u}(r)$ is a bounded monotonically increasing, left continuous function on $(0, 1]$ and right continuous at 0;
ii. $\overline{u}(r)$ is a bounded monotonically decreasing, left continuous function on $(0, 1]$ and right continuous at 0;
iii. $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$.

A crisp number r is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = r, 0 \le \alpha \le 1$. The set of all the fuzzy numbers is denoted by E^1.

A popular fuzzy number is the triangular fuzzy number $u = (u_m, u_l, u_r)$ where u_m denotes the modal value and the real values $u_l > 0$ and $u_r > 0$ represent the left and right fuzziness, respectively. Its parametric form is

$$\underline{u}(\alpha) = u_m + u_l(\alpha - 1), \qquad \overline{u}(\alpha) = u_m + u_r(1 - \alpha).$$

Triangular fuzzy numbers are fuzzy numbers in LR representation where the reference functions L and R are linear. The set of all triangular fuzzy numbers on $\mathbb{R}$ is called $\hat{FZ}$.

2.1. Operations on fuzzy numbers

We briefly mention fuzzy number operations defined by the extension principle [1]. Since coefficients vector of feedforward neural network is fuzzified in this paper, the operations we use in our fuzzy neural network are fuzzified by means of the extension principle. The h-level set of a fuzzy number X is defined by

$$[X]_h = \{x \in \mathbb{R} | \mu_X(x) \ge h\} \quad \text{for } 0 < h \le 1,$$

and $[X]_0 = \overline{\bigcup_{h\in(0,1]} [X]_h}$. Since level sets of fuzzy numbers become closed intervals, we denote $[X]_h$ by

$$[X]_h = [[X]_h^L, [X]_h^U],$$