Journal of Computational and Applied Mathematics 234 (2010) 233–239

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Almost periodic solutions for an impulsive delay Nicholson's blowflies model

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ARTICLE INFO

Article history:
Received 17 May 2009
Received in revised form 13 December 2009

MSC:
34K14
34K45

Keywords:
Almost periodic solution
Nicholson's blowflies model
Contraction mapping principle
Gronwall–Bellman's inequality

ABSTRACT

By means of the contraction mapping principle and Gronwall–Bellman's inequality, we prove the existence and exponential stability of positive almost periodic solution for an impulsive delay Nicholson's blowflies model. The main results are illustrated by an example.

1. Introduction

It has been recognized that impulsive delay differential equations (IDDEs for short) provide an adequate mathematical description for many real world phenomena [1–6]. Indeed, processes of models whose motions depend on the history as well as undergo abrupt changes in their states are best described by IDDEs. The qualitative properties of IDDEs have been extensively studied. The existence of periodic solutions, in particular, has occupied a great part of researchers' interest [7–14]. Although it is of great importance, however, the generalization to almost periodicity has been rarely considered in the literature. A few papers have dealt with the notion of almost periodicity for IDDEs; see [15–18].

In this paper, we consider one of the most popular models for population dynamics which is governed by a type of IDDEs. Indeed, sufficient conditions are established for the existence and the exponential stability of positive almost periodic solutions for an impulsive delay Nicholson's blowflies model. Our approach is based on the estimation of the Cauchy matrix of the corresponding linear impulsive differential equations.

2. Motivations and preliminaries

2.1. Motivations

In [19], Gurney et al. proposed the following nonlinear autonomous delay equation

$$x'(t) = -\alpha x(t) + \beta x(t-\tau)\mathrm{e}^{-\lambda x(t-\tau)}, \quad \alpha, \beta, \tau, \lambda \in (0, \infty) \tag{1}$$

to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [20]. Here $x(t)$ is the size of the population at time t, β is the maximum per capita daily egg production, $1/\lambda$ is the size at which the

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doi:10.1016/j.cam.2009.12.019

blowfly population reproduces at its maximum rate, α is the pair capita daily adult death rate and τ is the generation time. Eq. (1) is recognized in the literature as Nicholson's blowflies model. For more details on the dynamics behavior of this equation, see [21–23].

In the real world phenomena, the variation of the environment plays a crucial role in many biological and ecological dynamical systems. On the other hand, some dynamical systems are characterized by the fact that at certain moments in their evolution they undergo rapid changes in motions. Most notably this takes place due to certain seasonal effects such as weather, resource availability, food supplies, mating habits, etc. Thus, it is more appropriate to consider the generalized form of Nicholson's blowflies model

$$\begin{cases} x'(t) = -\alpha(t)x(t) + \sum_{i=1}^{n} \beta_i(t)x(t-\tau)e^{-\lambda_i(t)x(t-\tau)} + h(t), & t \neq \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k) + \delta_k, & k \in \mathbb{N}, \end{cases} \tag{2}$$

where

(i) $\alpha(t), \beta_i(t), \lambda_i(t), h(t) \in C[\mathbb{R}^+, \mathbb{R}^+]$, $\tau > 0$ and $\gamma_k, \delta_k \in \mathbb{R}$, $k \in \mathbb{N}$;
(ii) $\Delta x(t)$ denotes the difference $x(t^+) - x(t^-)$ where $x(t^+)$ and $x(t^-)$ define the limits from right and left, respectively;
(iii) θ_k represent the instants at which size of the population suffers an increment of δ_k units.

System (2) is popular enough among researchers. Indeed, it has been investigated by many authors who used different techniques to study the qualitative properties of its solutions. We name the papers [24–29] for continuous and discrete models of Nicholson's blowflies. The diffusive Nicholson's blowflies models have been studied in the papers [30–36]. One can easily see, nevertheless, that all equations investigated in the above-mentioned papers are under periodic assumptions and the existence of periodic solutions, in particular, has been under consideration. To the best of the author's knowledge, however, there is no published paper considering the notion of almost periodicity for model of Nicholson's blowflies. Motivated by this, the aim of this paper is to establish sufficient conditions for the existence and the exponential stability of positive almost periodic solution for Nicholson's blowflies model of form (2). We shall employ the contraction mapping principle and Gronwall–Bellman's inequality to prove our main results.

2.2. Preliminaries

Let $\{\theta_k\}_{k\in\mathbb{N}}$ be a fixed sequence such that $\sigma \leq \theta_1 < \theta_2 < \cdots < \theta_k < \theta_{k+1} < \cdots$ with $\lim_{k\to\infty} \theta_k = \infty$ and σ is a positive number. Denote by $PLC([\sigma-\tau, \sigma], \mathbb{R}^+)$ the space of all piecewise left continuous functions $\Psi : [\sigma-\tau, \sigma] \to \mathbb{R}^+$ with points of discontinuity of the first kind at $t = \theta_k$, $k \in \mathbb{N}$. By a solution of (2), we mean a function $x(t)$ defined on $[\sigma-\tau, \infty)$ and satisfying Eq. (2) for $t \geq \sigma$. Let $\xi \in PLC([\sigma-\tau, \sigma], \mathbb{R}^+)$, then Eq. (2) has a unique solution $x(t) = x(t; \sigma, \xi)$ defined on $[\sigma-\tau, \infty)$ and satisfies the initial condition

$$x(t; \sigma, \xi) = \xi(t), \quad \sigma - \tau \leq t \leq \sigma. \tag{3}$$

As we are interested in solutions of biological and ecological significance, we restrict our attention to positive ones.

To say that impulsive delay differential equations have positive almost periodic solutions, one need to adopt the following definitions of almost periodicity for such types of equations. The definitions are borrowed from [37].

Definition 1. The set of sequences $\{\theta_k^p\}$, $\theta_k^p = \theta_{k+p} - \theta_k$, $k, p \in \mathbb{N}$, is said to be uniformly almost periodic if for arbitrary $\varepsilon > 0$ there exists a relatively dense set of ε-almost periods common for any sequences.

Definition 2. A function $f \in PLC(\mathbb{R}^+, \mathbb{R}^+)$ is said to be almost periodic if the following conditions hold:

(a1) The set of sequences $\{\theta_k^p\}$ is uniformly almost periodic.
(a2) For any $\varepsilon > 0$ there exists a real number $\delta = \delta(\varepsilon) > 0$ such that if the points t' and t'' belong to the same interval of continuity of $f(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|f(t') - f(t'')| < \varepsilon$.
(a3) For any $\varepsilon > 0$ there exists a relatively dense set T of ε-almost periods such that if $\omega \in T$ then $|f(t+\omega) - f(t)| < \varepsilon$ for all $t \in \mathbb{R}^+$ satisfying the condition $|t - \theta_k| > \varepsilon$, $k \in \mathbb{N}$.

The elements of T are called ε-almost periods.

Related to Eq. (2), we consider the corresponding linear impulsive differential equation

$$\begin{cases} x'(t) = -\alpha(t)x(t), & t \neq \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k), & k \in \mathbb{N}. \end{cases} \tag{4}$$

In virtue of [37], it is well known that Eq. (4) with an initial condition $x(t_0) = x_0$ has a unique solution represented by the form

$$x(t; t_0, x_0) = X(t, t_0)x_0, \quad t_0, x_0 \in \mathbb{R}^+,$$