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The perturbed compound Poisson risk model with two-sided jumps

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ABSTRACT

In this paper, we consider a perturbed compound Poisson risk model with two-sided jumps. The downward jumps represent the claims following an arbitrary distribution, while the upward jumps are also allowed to represent the random gains. Assuming that the density function of the upward jumps has a rational Laplace transform, the Laplace transforms and defective renewal equations for the discounted penalty functions are derived, and the asymptotic estimate for the probability of ruin is also studied for heavy-tailed downward jumps. Finally, some explicit expressions for the discounted penalty functions, as well as numerical examples, are given.

1. Introduction

The classical compound Poisson risk model perturbed by a Brownian motion is given by:

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i + \sigma B(t), \tag{1.1}$$

where $u \geq 0$ is the initial surplus, $c > 0$ is the constant premium rate. The claim number process $\{N(t)\}_{t\geq 0}$ is a Poisson process with intensity $\lambda > 0$. The individual claims $X_1, X_2, \ldots$ are positive independent and identically distributed (i.i.d.) random variables. $\sigma > 0$ is the diffusion volatility and $\{B(t)\}_{t\geq 0}$ is a standard Brownian motion starting from zero. Finally $\{N(t)\}_{t\geq 0}$, $\{X_i\}_{i=1}^{\infty}$ and $\{B(t)\}_{t\geq 0}$ are assumed to be mutually independent.

Risk model (1.1) was first proposed in [1] to extend the classical compound Poisson risk model, in which the Brownian motion can be interpreted as random variability of the premiums income or the claims loss. Since then, many ruin problems associated with model (1.1) have been studied. For example, Dufresne and Gerber [2] studied the ruin probabilities by oscillation or by a claim. Gerber and Landry [3] examined the expected discounted penalty function of the deficit at ruin. Tsai [4] studied some discounted distributions for the surplus process perturbed by diffusion. For the same model, Tsai and Willmot [5,6] studied the discounted penalty functions and some (discounted) moments, respectively.

In this paper, we study a modified version of model (1.1). With all others being the same, the only change is to assume that the jumps are two-sided. The upward jumps can be explained to be the random gains of the company, while the downward jumps are interpreted as the random loss of the company. Denote the density of the jump X by f which is given by

$$f(x) = pf_+(x)I(x > 0) + qf_-(-x)I(x < 0), \tag{1.2}$$

where $p, q > 0, p+q = 1, I(A)$ is an indicator function of the event A, and f_+, f_- are two density functions on $[0, \infty)$. Let F_+, F_- denote the distributions, and μ_+, μ_- denote the means of f_+ and f_-. The risk model with two-sided jumps has been studied

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by many authors. Perry et al. [7] studied the joint distribution of the first exit time and overshoot (or undershoot) given that the distributions of the jumps were hyperexponential distribution or a special Coxian distribution. Kou and Wang [8] studied the Laplace transforms of the first passage time and the overshoot for a perturbed compound Poisson risk model where both the upward and downward jumps are exponentially distributed. Xing et al. [9] extended the results of Kou and Wang [8] to the case when the downward jumps are phase-type distributed and the upward jumps have an arbitrary distribution. Jacobsen [10] considered a more general risk model, the Markov-modulated diffusion risk model with two-sided jumps, and studied the Laplace transform of the time to ruin, the undershoot at ruin, as well as the probability of ruin. More recently, Yang and Zhang [11] studied a compound Poisson risk model with two-sided jumps by analyzing the discounted penalty function.

In this paper, we assume that the downward jumps have an arbitrary density function, while the density of the upward jumps has a rational Laplace transform $\hat{f}_-(s) = \int_0^\infty e^{-sx} f_-(x) dx$ given by

$$\hat{f}_-(s) = \frac{\nu(s)}{\prod_{i=1}^{m} (\nu_i + s)^{n_i}}, \tag{1.3}$$

where $m, n_i \in \mathbb{N}^+$ with $n_1 + n_2 + \cdots + n_m = n$, $\nu_i > 0$, $i = 1, 2, \ldots, m$, with $\nu_i \neq \nu_j$ for $i \neq j$. $\nu(s)$ is a polynomial function of degree $n - 1$ or less satisfying $\nu(0) = \prod_{i=1}^{m} \nu_i^{n_i}$. The rational family distributions, widely used in probability applications, include the Erlang, Coxian and phase-type distributions as special case, as well as mixture of them.

In what follows, $U(t)$ denotes the modified version of (1.1) with two-sided jumps described as above. Let $T = \inf\{t : U(t) \leq 0\}$ or ∞ otherwise, be the ruin time, and denote the ultimate ruin probability by

$$\psi(u) = Pr(T < \infty | U(0) = u).$$

To guarantee that ruin is possible, we assume that the following net profit condition holds, i.e.,

$$c + \lambda q \mu_- > \lambda p \mu_+. \tag{1.4}$$

Since ruin occurs immediately when $u = 0$, we have $\psi(0) = 1$. By observing the sample paths of $U(t)$, we know that ruin can be caused either by the oscillation of the Brownian motion or by a downward jump. Similar to Dufresne and Gerber [2] and Wang [12], we could decompose the probability of ruin as

$$\psi(u) = \psi_s(u) + \psi_d(u),$$

where $\psi_s(u)$ is the ruin probability when ruin is caused by a downward jump, and $\psi_d(u)$ is the ruin probability when ruin is caused by oscillation. We have the following initial conditions

$$\psi(0) = \psi_d(0) = 1, \qquad \psi_s(0) = 0. \tag{1.5}$$

Now for $\delta \geq 0$, define the (Gerber–Shiu) discounted penalty function by

$$\phi(u) = E[e^{-\delta T} w(U(T-), |U(T)|) I(T < \infty) | U(0) = u],$$

where $w(x_1, x_2)$, $x_1, x_2 \geq 0$, is a nonnegative function of the surplus before ruin $U(T-)$, and the deficit at ruin $|U(T)|$. Similarly, $\phi(u)$ can also be decomposed as

$$\phi(u) = \phi_s(u) + \phi_d(u),$$

where

$$\phi_s(u) = E[e^{-\delta T} w(U(T-), |U(T)|) I(T < \infty, U(T) < 0) | U(0) = u]$$

is the discounted penalty function at ruin that is caused by a downward jump, and

$$\begin{aligned} \phi_d(u) &= E[e^{-\delta T} w(U(T-), |U(T)|) I(T < \infty, U(T) = 0) | U(0) = u] \\ &= w(0, 0) E[e^{-\delta T} I(T < \infty, U(T) = 0) | U(0) = u] \end{aligned}$$

is the discounted penalty function at ruin that is caused by oscillation. Without loss of generality, we set $w(0, 0) = 1$, then the following initial conditions hold

$$\phi(0) = \phi_d(0) = 1, \qquad \phi_s(0) = 0. \tag{1.6}$$

In this paper, we study the discounted penalty functions for the perturbed compound Poisson risk model with two-sided jumps. The rest of this paper is organized as follows: In Section 2, we introduce a generalized Lundberg equation and analyze its roots on the right-half complex plane. By using these roots, the Laplace transforms for $\phi_s(u)$ and $\phi_d(u)$ are derived in Section 3. The defective renewal equations satisfied by $\phi_s(u)$ and $\phi_d(u)$ are derived in Section 4, and accordingly, the analytic expression for the discounted penalty functions are obtained. In Section 5, the asymptotic behavior for the probability of ruin is studied when the downward jumps are heavy-tailed. Finally, assuming that the density f_+ also has a rational Laplace transform, the explicit expressions for the discounted penalty functions as well as numerical examples are given.