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Using genetic algorithm to solve a new multi-period stochastic optimization model ☆

Xin-Li Zhang *, Ke-Cun Zhang

Faculty of Science, Xi'an Jiaotong University, Xi'an 710049, PR China

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ABSTRACT

This paper presents a new asset allocation model based on the CVaR risk measure and transaction costs. Institutional investors manage their strategic asset mix over time to achieve favorable returns subject to various uncertainties, policy and legal constraints, and other requirements. One may use a multi-period portfolio optimization model in order to determine an optimal asset mix. Recently, an alternative stochastic programming model with simulated paths was proposed by Hibiki [N. Hibiki, A hybrid simulation/tree multi-period stochastic programming model for optimal asset allocation, in: H. Takahashi, (Ed.) The Japanese Association of Financial Econometrics and Engineering, JAFFE Journal (2001) 89–119 (in Japanese); N. Hibiki A hybrid simulation/tree stochastic optimization model for dynamic asset allocation, in: B. Scherer (Ed.), Asset and Liability Management Tools: A Handbook for Best Practice, Risk Books, 2003, pp. 269–294], which was called a hybrid model. However, the transaction costs weren't considered in that paper. In this paper, we improve Hibiki's model in the following aspects: (1) The risk measure CVaR is introduced to control the wealth loss risk while maximizing the expected utility; (2) Typical market imperfections such as short sale constraints, proportional transaction costs are considered simultaneously. (3) Applying a genetic algorithm to solve the resulting model is discussed in detail. Numerical results show the suitability and feasibility of our methodology.

1. Introduction

Rational investors maximize the expected utility of return from their investment portfolio, or minimize their risk exposure of return, subject to their required expected return. They must decide on their optimal portfolio in securities in order to meet their satisfaction. This paper discusses optimal dynamic investment policies for investors who make an investment decision in each asset category over time. This problem is called "dynamic asset allocation". The choice of an optimal portfolio of assets has been a major research topic in financial economics. Financial economists have understood at least since the early work of [1,2] that the solution to a multiperiod portfolio choice problem can be very different from the solution to a static portfolio choice problem. In particular, [3,4] and others had shown that in general shiftng investment opportunities could have important effects on optimal portfolios for investors with long horizons. Many empirical researches (see, for example, [5,18]) had shown that expected asset returns seem to vary through time, so that investment opportunities were not constant.

Asset allocation decisions are critical for investors with diversified portfolios. Institutional investors must manage their strategic asset mix over time to achieve favorable returns, in the presence of uncertainties and subject to various legal

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* Corresponding author.
E-mail address: zhgylgp@gmail.com (X.-L. Zhang).

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constraints, policies, and other requirements. A multi-period portfolio optimization model can be used in order to determine an optimal asset mix. The concept of scenarios is typically employed for modeling random parameters in multi-period stochastic programming models. Scenarios are constructed via a tree structure (see [6,7] for detailed discussions). The model is based on the expansion of the decision space, taking into account a conditional nature of the scenario tree. Conditional decisions are made at each node, subject to the modeling constraints. To ensure that the constructed representative set of scenarios covers the set of possibilities to a sufficient degree, the numbers of decision variables and constraints in the scenario tree may grow exponentially. This model is called a scenario tree model. Recently, an alternative stochastic programming model using simulated paths was proposed in [8]. Hibiki [9] developed a general formulation for several investment strategies, and highlighted its characteristics and properties by using some numerical tests. Scenarios are constructed via a simulated path structure. We can generate sample paths associated with asset returns using a Monte Carlo simulation method. The advantage of the simulated path structure compared to the tree structure is to give a better accuracy to describe uncertainties of asset returns. The model not only describes the uncertainties on the simulated path structure but also makes conditional decisions on the tree structure. Therefore, it is called a "hybrid" model. It can be easily implemented and efficiently solved using a standard mathematical programming software package. The hybrid model is developed to overcome the shortcoming of the scenario tree model associated with uncertainties. Hibiki [10] answered the question that how quantitatively the hybrid model was better than the scenario tree model, which was not shown in the previous papers [8,9] and compared the two types of multi-period stochastic optimization models, and clarified that the hybrid model can evaluate and control risk better than the scenario tree model by some numerical tests.

As we know, a consumer who ignores realistic transaction costs, and trades continuously, would end up bankrupt. Balduzzi and Lynch [11] found that both the losses in utility for behaving myopically and ignoring asset return predictability can be substantial, and that ignoring realistic transaction costs imposes significant utility costs that range from 0.8% up to 16.9% of wealth. Unfortunately, studies that incorporate transaction costs typically assume that the opportunity set is constant through time. For example, by imposing a constant opportunity set, Constantinides and Schroder [12,13] found that transaction costs affect the portfolio choice, since the optimal policy was a no-trade region with return to the nearer boundary (for proportional transaction costs) or inside the boundary (for fixed transaction costs) when rebalancing. On the other hand, studies that examine the impact of return predictability on portfolio selection usually do not consider transaction costs. Typical researches of this kind are [14–16]. Most papers in this class described the return predictability as a vector autoregression (VaR) process. The VaR description only considers time variation in the first moments of asset returns. But volatility plays an important role in much of the modern finance theory, empirical evidence shows that the volatilities of most risky assets do not change through time. In practice, to ensure that the investor has enough amount of wealth at the horizon, it is necessary to control the wealth loss risk so that, with some probability level, the maximum wealth loss does not exceed some critical value. Therefore, even an economic agent's primary objective in the standard financial economics paradigms is the expected utility maximization, we feel that it is worthwhile to directly embed the risk management objective into a utility maximizing framework, or at least to assume that agents may limit their risks while maximizing expected utility. For risk management in this context, one possible choice is the Value-at-Risk (VaR) based risk control, since VaR exactly describes the loss that can occur over a given period, at a given confidence level, due to exposure to market risk. Unfortunately, recent studies (see [17], for example) showed that VaR was not an acceptable, correct risk measure because of the following: it did't measure losses exceeding VaR, VaR was not subadditive, and so on. Recognizing the shortcomings of VaR to stem from its focus on the probability of a loss, regardless of the magnitude, the conditional VaR (CVaR), defined as the expected value of the losses exceeding VaR, can be adopted instead. Except for being a correct, coherent risk measure and overcoming the VaR's shortcomings, the main advantage of CVaR is that it can be easily computed and embedded in the portfolio selection problem. Consequently, CVaR could be used to better control the agent's wealth loss risk. Zhao and Ziemba [19] proposed a new stochastic control model for dynamic asset allocation problems. The model maximized the expected terminal wealth while controlling for downside risk. Although the return/risk performance comparison with VaR as the risk measure showed that the found optimal strategy had superior performance, the proposed optimization model could be analytically solved only if admissible controls were restricted to the set of all risk-neutral excess return strategies and the stock prices were lognormally distributed. For general cases, the Monte Carlo simulation or heuristic method had to be used, which was computational demanding and slow in convergence. Moreover, setting under a complete market, market frictions such as transaction costs were not considered in Zhao and Ziemba (2000). From above-mentioned we know that the transaction costs play a crucial role in transaction. But in the existing work [9,10], it didn't consider the transaction costs and just gave a heuristic algorithm to solve the Hybrid model, and could not guarantee to derive the global optimal solutions to the fixed-proportion strategy.

In this paper, we not only use CVaR to control the risk, but also the typical market imperfections such as short sale constraints, proportional transaction costs are considered, and thus obtain a new stochastic optimization model. In order to give the difference between the Hybrid model and the model proposed in this paper, we apply the simulated path proposed in [10]. Also, we give a genetic algorithm to solve the Hybrid model and the new one in this paper. Numerical results show that our model can evaluate and control risk better than the Hybrid model. The process of how to solve the resulting model is discussed in detail.

The remainder of the paper is organized as follows: Section 2 presents the concept and formulations of the two kinds of models, and provides the detailed process of how to construct the new model. In Section 3, we propose how to use a genetic algorithm to solve the two portfolio models and some numerical tests, which are given to verify the effective and efficient of