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Modeling fuzzy multi-period production planning and sourcing problem with credibility service levels

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ABSTRACT

A great deal of research has been done on production planning and sourcing problems, most of which concern deterministic or stochastic demand and cost situations and single period systems. In this paper, we consider a new class of multi-period production planning and sourcing problem with credibility service levels, in which a manufacturer has a number of plants and subcontractors and has to meet the product demand according to the credibility service levels set by its customers. In the proposed problem, demands and costs are uncertain and assumed to be fuzzy variables with known possibility distributions. The objective of the problem is to minimize the total expected cost, including the expected value of the sum of the inventory holding and production cost in the planning horizon. Because the proposed problem is too complex to apply conventional optimization algorithms, we suggest an approximation approach (AA) to evaluate the objective function. After that, two algorithms are designed to solve the proposed production planning problem. The first is a PSO algorithm combining the AA, and the second is a hybrid PSO algorithm integrating the AA, neural network (NN) and PSO. Finally, one numerical example is provided to compare the effectiveness of the proposed two algorithms.

1. Introduction

Production planning is viewed as the plans and arrangements of the production mission and progress in production scheduled time. In recent years, production planning – especially uncertain production planning – has been studied widely in the field of production planning management. Galbraith [1] defined uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. In the real world, there are many forms of uncertainty that affect production processes. Ho [2] categorized them into two groups: (i) environmental uncertainty and (ii) system uncertainty. Environmental uncertainty included uncertainties beyond the production process such as demand uncertainty and supply uncertainty. System uncertainty was related to uncertainties within the production process such as operation yield uncertainty, production lead time uncertainty, quality uncertainty, failure of production system and change to product structure. Uncertainty can be present as randomness and fuzziness in the production environment. This uncertainty will result in more realistic production planning models. However, the inclusion of uncertainty in the production system parameters is a more difficult task in terms of modeling and solving. Over the years, there has been much research and many applications with the aim of modeling the uncertainty in production planning problems, such as the material requirements planning (MRP) model [3,4], the hierarchical production planning (HPP) model [5,6], the aggregate production planning (APP) model [7–9], the supply chain (SC) model [10,11] and other well-known production planning models in the literature [12–15].

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In order to handle probabilistic uncertainty in the production decision systems, some meaningful stochastic production planning models have been proposed in the literature such as [12]. They dealt with a stochastic production planning problem with service level requirements, and provided non-sequential and deterministic equivalent formulations of the model. Kelly, Clendenen and Dardeau [16] extended the economic lot scheduling problem for the single-machine multi-product case with random demands. Their objective was to find the optimal length of production cycles that minimizes the sum of set-up costs and inventory holding costs per unit of time, and satisfy the demand of products at the required service levels. Zäpfel [14] claimed that MRP II systems could be inadequate for the solution of production planning problems with uncertain demand because of the insufficiently supported aggregation/disaggregation process. The paper then proposed a procedure to generate an aggregate plan and a consistent disaggregate plan for the Master Production Schedule.

In fuzzy decision systems, fuzzy production planning models have been considered by many researchers. Based on fuzzy set theory and possibility theory [17–20], many researchers applied them to fuzzy optimization models such as [7,9,21,22]. Among them, Wang and Fang [7] presented a fuzzy linear programming model for solving the aggregate production planning problem with multiple objectives. Gen and Tsujimura [9] proposed a fuzzy model with multiple objectives for aggregate planning, with objective function coefficients, technological coefficients, and resource right-hand side constraints represented by triangular fuzzy numbers. Tanaka et al. [21] transformed possibilistic linear programming problems based on exponential possibility distributions into non-linear optimization problems. In order to solve optimization problems easily, algorithms for obtaining center vectors and distribution matrices in sequence were proposed. Shih [22] resolved the cement transportation planning problem using fuzzy linear programming methods. Three types of fuzzy linear programming models were used to determine the optimal transportation amount and the capacity of new facilities.

The purpose of this paper is to present a realistic production planning model. We take credibility theory [23–25] as the theoretical foundation of fuzzy optimization and develop a multi-period production planning and sourcing problem with a credibility service levels model, in which demands and costs are uncertain and assumed to be fuzzy variables with known possibility distributions. The objective of the problem is to minimize the total expected cost, including the expected value of the sum of the inventory holding and production cost in the planning horizon. Also, we transform the credibility constraint into its crisp equivalent form when demands are independent normal fuzzy variables. Then, we suggest an AA to evaluate the objective function. Since the approximating production planning problem is neither linear nor convex, conventional optimization algorithms cannot be applied. Therefore, two approximation-based algorithms are designed to solve the proposed production planning problem. The first is the PSO algorithm which integrates the AA [26] and PSO [27,28], and the second is the hybrid PSO algorithm which combines the AA, neural network (NN) and PSO. One numerical example is also provided to compare the effectiveness of the two algorithms.

The rest of this paper is organized as follows. In Section 2, we recall some preliminary knowledge. Section 3 proposes a new class of fuzzy production planning model. In Section 4, we employ the AA to discretize the objective function of the fuzzy production planning model, and deal with the convergence of the AA. The convergent result allows us to design two approximation-based PSO algorithms to solve the proposed fuzzy production planning problem in Section 5, and one numerical example is provided in this section to compare the effectiveness of the two algorithms. Section 6 summarizes the main results in this paper.

2. Preliminaries

Given a universe Γ, $\mathcal{P}(\Gamma)$ is the power set of Γ and Pos is a set function defined on $\mathcal{P}(\Gamma)$. Let ξ be a fuzzy variable with membership function $\mu(x)$ and r a real number. Then the possibility measure of a fuzzy event $\{\xi \le r\}$ is defined as

$$\mathrm{Pos}\{\xi \le r\} = \sup_{x\le r} \mu(x)$$

for any real number r.

The credibility measure [23] of the fuzzy event $\{\xi \le r\}$ was defined as

$$\mathrm{Cr}\{\xi \le r\} = \frac{1}{2}\left(1 + \sup_{x\le r}\mu(x) - \sup_{x>r}\mu(x)\right)$$

for any real number r.

Using credibility measure, the expected value of the fuzzy variable ξ, denoted by $E[\xi]$, was defined as

$$E[\xi] = \int_0^{\infty} \mathrm{Cr}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \le r\}\mathrm{d}r$$

provided that at least one of the two integrals is finite.

In particular, if ξ is a finite discrete fuzzy variable with the following membership function

$$\mu_\xi(x) = \begin{cases} \mu_1, & \text{if } x = \widehat{\xi}_1 \\ \mu_2, & \text{if } x = \widehat{\xi}_2 \\ \cdots & \\ \mu_n, & \text{if } x = \widehat{\xi}_n \end{cases}$$