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Non-stationary problem of active sound control in bounded domains

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ABSTRACT

The present paper deals with the non-stationary problem of active shielding of a domain from undesirable external sources of noise. Active shielding is achieved by constructing additional (secondary) sources in such a way that the total contribution of all sources leads to the desirable effect. The problem is formulated as an inverse source problem with the secondary sources positioned outside the domain to be shielded. Along with the undesirable field (noise) to be shielded the presence of a desirable component ("friendly" sound) is accepted in the analysis. The constructed solution of the problem requires only the knowledge of the total field (noise) on the perimeter of the shielded domain. Some important aspects of the problem are addressed in the paper for the first time, such as the non-stationary formulation of the problem, existence of the resonance regimes and sensitivity of the solution to the input errors. The obtained solution is applicable to aeroacoustics problems.

0. Introduction

The active shielding (AS) of some domains from the effect of the external field (noise) generated in other domains is achieved by constructing additional (secondary) sources in such a way that they cancel each other. Along with the undesirable (noise) sources situated outside the domain to be shielded, the existence of internal (desirable) sources is accepted in the analysis. The problem can be formulated as an inverse source problem. The reviews of some theoretical and experimental methods related to the problem subjects can be found in [1,2]. Most theoretical approaches assume that rather detailed information about the undesirable sources and the properties of the medium is known. Unlike these approaches, the JMC method [3,2], based on Huygens' construction, requires only the information on the undesirable field on the perimeter of the shielded domain. However, it has not yet been used in the case when the desirable field ("friendly" sound), generated in the shielded domain, needs to be taken into account. In addition, the JMC method, as many others, has only been used for problems formulated in unbounded domains.

The Difference Potential Method (DPM), proposed in [4,5], allows us to circumvent many limitations of other approaches in that it requires much less information of the total noise field. For example, in [6,4], the solution of the problem is obtained by this method in a finite-difference formulation and requires only the knowledge of the total field (both desirable and undesirable) at the mesh boundary of the shielded domain; any other information on the sources and medium is not required. One can say that the solution procedure uses the minimum information *a priori* available. In [7], the DPM method was applied to the Helmholtz equation. The questions of optimization for this equation were comprehensively studied by Lončarić and Tsynkov; see, e.g., [8].

The DPM-based solution was extended by Ryaben'kii and Utyuzhnikov to arbitrary hyperbolic systems of equations, including the Euler acoustics equations with constant and variable coefficients, in [9]. In [10], the authors considered the AS problem in bounded domains, showed that the control sources are capable of not disturbing even the echo of the "friendly"

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sound component and explained the mechanism of this property of the solution. For the system of first-order equations in continuous spaces, the AS solution was first obtained in [11] for time-harmonic waves under rather general assumptions. The DPM-based discrete solution was shown to approach to this continuous solution as the spatial mesh is refined. The secondary single-layer term is obtained for the Euler acoustics equations.

The solution [11] is also applicable to higher-order equations by means of reducing them to a set of first-order equations. In particular, the Helmholtz equation is considered in this way. The AS solution is represented by a linear combination of single- and double-layer potentials and coincides with the solution first obtained in [7]. The general statement of essentially non-stationary problem (broad band) is first addressed in [12]. The solution of the AS inverse problem is obtained in both linear non-stationary and nonlinear stationary formulations. In [13], the problem of AS in composite domains is formulated and its general solution is provided in the general finite-difference formulation. The counterpart of the problem in continuous spaces is solved in [14]. The principal novelty of the problem considered in [13] is that it allows a selective communication between different sub-domains. The solution of the problem is constructed by means of a predictor–corrector algorithm.

In the present work, the AS solution is considered for the general formulation of the wave equation in the time domain. It is demonstrated that the solution is applicable even to resonance regimes. Since in practical applications errors in the input and output parameters are unavoidable, the sensitivity analysis on the noise cancelation is given for the wave equation. The application to aeroacoustics is also considered.

1. General formulation of the AS problem

First, let us introduce some domain D: $\overline{D} \subseteq \mathbb{R}^m$ with smooth boundary Γ_0 and a sub-domain D^+: $\overline{D^+} \subset D$, having smooth boundary Γ.

Let us assume that some field (sound) U is described by the following boundary value problem (BVP):

$$LU = f, \tag{1}$$

$$U \in \Xi_D. \tag{2}$$

Here, the operator L is a linear differential operator, Ξ_D is some functional space specified further. In particular, the operator L can correspond to the acoustics equations. It is supposed that BVP (1), (2) is well-posed for any right-hand side f: $f \in L_2^{\text{loc}}(D)$. Thereby, the boundary conditions are supposed to be implicitly included in the definition of the space Ξ_D. To avoid any possible confusion, we assume that the boundary conditions are homogeneous and formulated locally on the boundary Γ_0.

We assume that the sources on the right-hand side can be placed both on D^+ and outside D^+:

$$f = f^+ + f^-, \tag{3}$$

$$\operatorname{supp} f^+ \subset D^+, \quad \operatorname{supp} f^- \subset D^- \stackrel{\text{def}}{=} D \setminus \overline{D^+}.$$

Thus, f^+ are "friendly" field (sound) sources, while f^- generates an "adverse" field (noise).

Suppose that we know the trace of the function U on the boundary Γ of the domain D^+. We note that only this information is assumed to be available. In particular, the distribution of the sources f on the right-hand side is unknown. The AS problem is then reduced to finding additional sources G in $\overline{D^-}$ (see Fig. 1) such that the solution of the following BVP

$$LV = f + G, \tag{4}$$

$$\operatorname{supp} G \subset \overline{D^-},$$

$$V \in \Xi_D$$

coincides on the domain D^+ with the solution of BVP (1), (2) if $f^- \equiv 0$:

$$LU^+ = f^+, \tag{5}$$

$$U^+ \in \Xi_D.$$

Thus, we seek a source term G such that on the domain D^+ the functions U and V coincide with each other: $V_{D^+} = U^+_{D^+}$.

2. Non-stationary AS problem

Suppose that the field U is the solution of a well-posed initial-boundary value problem (IBVP) in the cylinder $K_T = D \times (0, T) \subseteq \mathbb{R}^{m+1} (T > 0)$:

$$LU \stackrel{\text{def}}{=} L_t^{(p)} U + \sum_1^m A^i \frac{\partial U}{\partial x^i} = f, \tag{6}$$

$$U \in \Xi_D, \tag{7}$$