Journal of Computational and Applied Mathematics 234 (2010) 1904–1911

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Time-harmonic solution for acousto-elastic interaction with controllability and spectral elements

Sanna Mönkölä

Department of Mathematical Information Technology, University of Jyväskylä, P.O. Box 35 (Agora), FI-40014 University of Jyväskylä, Finland

ARTICLE INFO

Article history:
Received 14 December 2007
Received in revised form 25 March 2008

MSC:
74F10
65N30
65N35
93B05

Keywords:
Fluid-structure interaction
Acoustic waves
Elastic waves
Coupled problem
Time-harmonic solution
Spectral element method
Controllability
Conjugate gradient algorithm

ABSTRACT

The classical way of solving the time-harmonic linear acousto-elastic wave problem is to discretize the equations with finite elements or finite differences. This approach leads to large-scale indefinite complex-valued linear systems. For these kinds of systems, it is difficult to construct efficient iterative solution methods. That is why we use an alternative approach and solve the time-harmonic problem by controlling the solution of the corresponding time dependent wave equation.

In this paper, we use an unsymmetric formulation, where fluid-structure interaction is modeled as a coupling between pressure and displacement. The coupled problem is discretized in space domain with spectral elements and in time domain with central finite differences. After discretization, exact controllability problem is reformulated as a least-squares problem, which is solved by the conjugate gradient method.

1. Introduction

Acoustic waves are small oscillations of pressure, which are associated with local motions of particles in fluid domain Ω_f. The linear theory of elasticity models mechanical properties in structure Ω_s assuming small deformations. Acousto-elastic interaction between these two media constitutes a coupled problem. Several phenomena, such as seismic waves in the earth and ultrasonic waves used to detect flaws in materials, can be described by an acousto-elastic model. Two approaches, in which the displacement is solved in the elastic structure, predominate in modeling the interaction between acoustic and elastic waves. Expressing the acoustic wave equation by the velocity potential results in a symmetric system of equations (see, e.g., [1–4]), while using the pressure in the fluid domain leads to an unsymmetric formulation (see, e.g., [5–8]).

In this paper, we present the acousto-elastic interaction between pressure and displacement, and thereby concentrate on the unsymmetric approach. We formulate the time-harmonic acousto-elastic interaction as an exact controllability problem [9] via the corresponding time dependent system. The time dependent problem is discretized in space domain with the spectral element method (SEM) and in time domain with the second-order central finite differences. The combination of these discretization methods is well known with wave equations (see, e.g., [10]). The methods related to spectral elements are studied in the context of the time dependent acousto-elastic problem with second-order time-stepping schemes; see for instance Refs. [11,12,6].

After discretization, we solve the control problem by a conjugate gradient (CG) algorithm which is related to that developed in [13] for the acoustic wave equation. If an unpreconditioned CG algorithm is used, the number of iterations

E-mail address: sanna.monkola@jyu.fi.

0377-0427/$ – see front matter
doi:10.1016/j.cam.2009.08.040

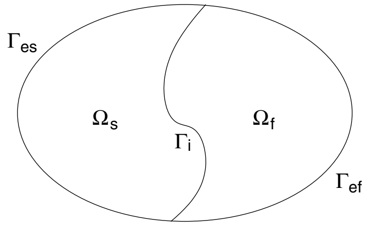

Fig. 1. The domain Ω is divided into the solid part Ω_s and the fluid part Ω_f.

grows rapidly with the order of spectral element [14]. That is why we use a modification of Kickinger's [15] algebraic multigrid (AMG), introduced in [16], for preconditioning the conjugate gradient algorithm.

The rest of this paper is organized as follows. First, the mathematical model is presented in Section 2. Then, we discretize the coupled problem in space domain with spectral elements in Section 3. For time discretization we use central finite differences in Section 4. In Section 5, we present the control problem and the preconditioned conjugate gradient algorithm. Finally, we show some numerical experiments in Section 6.

2. Mathematical model

We consider the use of a control algorithm to solve the time-harmonic acousto-elastic problem in the domain $\Omega \subset \mathbb{R}^2$, which is divided into the solid part Ω_s and the fluid part Ω_f by the interface Γ_i (see Fig. 1). Instead of solving directly the time-harmonic equation, we return to the corresponding time dependent equation (see, e.g., [17,10]) and look for time-periodic solution. The convergence is accelerated with a control technique by representing the original time-harmonic equation as an exact controllability problem [18,19] for the time dependent wave equation

$$\frac{1}{\rho_f(\mathbf{x})c(\mathbf{x})^2}\frac{\partial^2 p_f}{\partial t^2} - \nabla\cdot\left(\frac{1}{\rho_f(\mathbf{x})}\nabla p_f\right) = f, \quad \text{in } \Omega_f \times [0,T], \tag{1}$$

$$p_f = 0, \quad \text{on } \Gamma_{0\mathrm{f}} \times [0,T], \tag{2}$$

$$\frac{1}{c(\mathbf{x})}\frac{\partial p_f}{\partial t} + \frac{\partial p_f}{\partial \mathbf{n}_f} = y_{\mathrm{ext}}, \quad \text{on } \Gamma_{\mathrm{ef}} \times [0,T], \tag{3}$$

$$\rho_f(\mathbf{x})\frac{\partial^2 \mathbf{u}_s}{\partial t^2}\cdot\mathbf{n}_s - \frac{\partial p_f}{\partial \mathbf{n}_f} = 0, \quad \text{on } \Gamma_{\mathrm{i}} \times [0,T], \tag{4}$$

$$\rho_s(\mathbf{x})\frac{\partial^2 \mathbf{u}_s}{\partial t^2} - \nabla\cdot\sigma(\mathbf{u}_s) = \mathbf{f}, \quad \text{in } \Omega_s \times [0,T], \tag{5}$$

$$\mathbf{u}_s = 0, \quad \text{on } \Gamma_{0\mathrm{s}} \times [0,T], \tag{6}$$

$$\rho_s(\mathbf{x})\mathbf{B}\frac{\partial \mathbf{u}_s}{\partial t} + \sigma(\mathbf{u}_s)\mathbf{n}_s = \mathbf{g}_{\mathrm{ext}}, \quad \text{on } \Gamma_{\mathrm{es}} \times [0,T], \tag{7}$$

$$\sigma(\mathbf{u}_s)\mathbf{n}_s - p_f\mathbf{n}_f = 0, \quad \text{on } \Gamma_{\mathrm{i}} \times [0,T], \tag{8}$$

where f, y_{ext}, $\mathbf{f}$, and $\mathbf{g}_{\mathrm{ext}}$ are the source terms. Length of the time interval is marked as T, p_f denotes the pressure, and $\mathbf{u}_s = (\mathbf{u}_{s1}, \mathbf{u}_{s2})^{\mathrm{T}}$ is the displacement field depending on the spatial variable $\mathbf{x} = (x_1, x_2)^{\mathrm{T}} \in \mathbb{R}^2$. Coefficients $\rho_f(\mathbf{x})$ and $\rho_s(\mathbf{x})$ represent the densities of media in domains Ω_f and Ω_s, respectively, and $c(\mathbf{x})$ is the speed of sound in fluid domain. The stress tensor is expressed as $\sigma(\mathbf{u}_s) = \rho_s(\mathbf{x})\left(c_p(\mathbf{x})^2 - 2c_s(\mathbf{x})^2\right)(\nabla\cdot\mathbf{u}_s)\mathcal{I} + 2\rho_s(\mathbf{x})c_s(\mathbf{x})^2\epsilon(\mathbf{u}_s)$ with the speed of the pressure wave $c_p(\mathbf{x})$, the speed of the shear wave $c_s(\mathbf{x})$, the identity matrix $\mathcal{I}$, and the linearized strain tensor $\epsilon = \frac{1}{2}\left(\nabla\mathbf{u}_s + (\nabla\mathbf{u}_s)^{\mathrm{T}}\right)$. The outward normal vectors to domains Ω_f and Ω_s are marked as $\mathbf{n}_f = (n_{f1}, n_{f2})^{\mathrm{T}}$ and $\mathbf{n}_s = (n_{s1}, n_{s2})^{\mathrm{T}}$.

The fluid domain is bounded by $\Gamma_{\mathrm{f}} = \Gamma_{0\mathrm{f}} \bigcup \Gamma_{\mathrm{ef}} \bigcup \Gamma_i$, and $\Gamma_{\mathrm{s}} = \Gamma_{0\mathrm{s}} \bigcup \Gamma_{\mathrm{es}} \bigcup \Gamma_i$ constitutes the boundary for the solid domain Ω_s. The boundaries $\Gamma_{0\mathrm{f}}$ and $\Gamma_{0\mathrm{s}}$ are assumed to be rigid, whereas on the artificial boundaries Γ_{ef} and Γ_{es} we impose the conventional first-order absorbing boundary conditions [20,21], where $\mathbf{B}$ is a symmetric positive definite 2×2-matrix defined by

$$\mathbf{B} = \begin{pmatrix} c_p(\mathbf{x})n_{s1}^2 + c_s(\mathbf{x})n_{s2}^2 & n_{s1}n_{s2}(c_p(\mathbf{x}) - c_s(\mathbf{x})) \\ n_{s1}n_{s2}(c_p(\mathbf{x}) - c_s(\mathbf{x})) & c_p(\mathbf{x})n_{s2}^2 + c_s(\mathbf{x})n_{s1}^2 \end{pmatrix}.$$

In addition to the system (1)–(8), we take into account the initial conditions $\mathbf{e} = (\mathbf{e}_0, \mathbf{e}_1)^{\mathrm{T}}$ such that $\mathbf{e}_0 = (\mathbf{e}_{0f}, \mathbf{e}_{0s})^{\mathrm{T}}$ and $\mathbf{e}_1 = (\mathbf{e}_{1f}, \mathbf{e}_{1s})^{\mathrm{T}}$, and

$$p_f(\mathbf{x}, 0) = \mathbf{e}_{0f}, \quad \frac{\partial p_f}{\partial t}(\mathbf{x}, 0) = \mathbf{e}_{1f}, \quad \text{in } \Omega_f, \tag{9}$$