Journal of Computational and Applied Mathematics 230 (2009) 135–142

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A cut-peak function method for global optimization[☆]

Yuncheng Wang[a], Weiwu Fang[b,*], Tianjiao Wu[b]

[a] *College of Information Sciences and Engineering, Shandong Agricultural University, Taian 271018, China*

[b] *Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, 100080, China*

ARTICLE INFO

Article history:
Received 1 December 2007
Received in revised form 30 October 2008

MSC:
90C30
90C59

Keywords:
Global optimization
Cut-peak function
Box constrained minimization problem
Smoothing technique

ABSTRACT

A new method is proposed for solving box constrained global optimization problems. The basic idea of the method is described as follows: Constructing a so-called cut-peak function and a choice function for each present minimizer, the original problem of finding a global solution is converted into an auxiliary minimization problem of finding local minimizers of the choice function, whose objective function values are smaller than the previous ones. For a local minimum solution of auxiliary problems this procedure is repeated until no new minimizer with a smaller objective function value could be found for the last minimizer. Construction of auxiliary problems and choice of parameters are relatively simple, so the algorithm is relatively easy to implement, and the results of the numerical tests are satisfactory compared to other methods.

1. Introduction

Recently, due to increasing demand in some areas of science and engineering, there has been considerable interest in the study of implementable algorithms for finding a global minimizer of multimodal functions. Generally speaking, the reported approaches can be put into two categories: deterministic and probabilistic; moreover, deterministic methods are usually more efficient than probabilistic methods, such as the filled function method with remarkable advances [1–4] and the tunneling algorithm [5,6].

In this paper, a deterministic algorithm, which is referred to as the *cut-peak method (briefly called C-P method)*, is proposed for finding a global minimizer of box constrained global optimization problems. In this method, an auxiliary problem is introduced at a local minimizer of the original problem; more precisely, the auxiliary problem is the minimization problem of a so-called cut-peak function and the objective function of the original problem, and the present local minimum point of the original problem becomes the unique maximum point of the auxiliary problem in the constraint region.

For a local minimum solution of both an auxiliary problem and the original problem this procedure is repeated until no other solution giving smaller objective function value could be found. Similar to the filled function method and the tunneling method, the method proposed in this paper consists of two main phases: the phase of finding a local minimizer and the phase of finding a different minimizer in another valley; but the designs of the three methods are different — the filled function method is based on some function perturbations, which are used to prevent searching from bogging down at a local valley, tunneling method is on solving inequality systems [7], and the new method is on cutting peaks at a local minimizer point, which leads into a choice function instead of the original objective function — so that the adopted functions and their features

☆ This work has been supported by the National Key Basic Research Project of China (NKBRPC) 2004CB318000 and Chinese NSF under the grant 90103033; The authors gratefully thank the anonymous reviewers for their many valuable suggestions.

* Corresponding author.
E-mail addresses: ycwang@sdau.edu.cn (Y. Wang), fangww@amss.ac.cn (W. Fang).

0377-0427/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2008.10.069

are different. In the new method, the construction of auxiliary problems and choice of parameters are relatively simple, thus, the implementation of the algorithm is easier than others.

The rest of this paper is organized as follows. The basic concepts and notations are given in Section 2, and a principal algorithm and convergence analysis are presented in Section 3. Because the auxiliary function (the choice function) is non-differentiable at the points where the cut-peak function and the objective function meet, a smoothing technique is employed to approximate the auxiliary function in Section 4, and an implementable algorithm and preliminary numerical tests with 10 problems are also shown in this section.

2. The cut-peak function and choice function

2.1. Definitions of cut-peak function and choice function

Consider the global optimization problem

$$\min_{x\in\Omega} f(x) \tag{P}$$

where $f(x)$ is continuously differentiable and defined on a compact region $\Omega = \{x \in R^n : a \le x \le b\} \subset R^n$, a and b are two given vectors in R^n.

Definition 2.1.1. $w(r, x^{(k)}, x)$ is called a *cut-peak function* of $f(x)$ at point $x^{(k)}$ with a positive parameter r (sometimes a vector) if the following two conditions are satisfied:

(i) $x^{(k)}$ is the unique maximum point of $w(r, x^{(k)}, x)$, and $w(r, x^{(k)}, x^{(k)}) = f(x^{(k)})$;
(ii) for any direction $d \in R^n$, $w(r, x^{(k)}, x^{(k)} + \lambda d)$ is strictly decreasing with respect to step length λ, and

$$\lim_{\lambda\to+\infty} w(r, x^{(k)}, x^{(k)} + \lambda d) = f(x^{(k)}) - c(r) > -\infty,$$

where $c(r)$ is a positive scalar with respect to given constant r and is called the *maximum cut* of $w(r, x^{(k)}, x)$ at $x^{(k)}$.

Remark 2.1. In practice, r or $c(r)$ can be regarded as a given constant depending on the accuracy required and is independent of $x^{(k)}$.

The cut-peak function can be formulated in different ways. A suggested formulation is as follows:

$$w(r, x^{(k)}, x) = f(x^{(k)}) - f_0(r, \|x - x^{(k)}\|),$$

where $f_0(r, \cdot)$ satisfies the following conditions:

(i) $f_0(r, \cdot)$ is strictly increasing from R^+ to R^+,
(ii) $f_0(r, 0) = 0$, and
(iii) $\lim_{t\to+\infty} f_0(r, t) = c(r) < +\infty$, i.e., $f_0(r, \cdot)$ is upper bounded.

The following are two concrete examples on the $f_0(r, \cdot)$:

$$f_0(r, t) = \frac{rt^2}{1 + t^2}$$

and

$$f_0(r, t) = r(1 - \mathrm{e}^{-t^2}).$$

Definition 2.1.2. $F(r, x^{(k)}, x) = \min(f(x), w(r, x^{(k)}, x))$ is called a choice function of $f(x)$ crossing through the point $x^{(k)}$.

An example of an objective function, cut-peak function and choice function is shown in Fig. 1: the broken line means an objective function ($f(x) = -\mathrm{e}^x \sin(2\pi x)$ where $x \in [0, 4]$ and $x^{(k)} = 1.275$), the dotted line means a cut-peak function ($w(r, x^{(k)}, x) = f(x^{(k)}) - \frac{5(x-x^{(k)})^2}{1+(x-x^{(k)})^2}$ where $x \in [0, 4]$ and $x^{(k)} = 1.275$), and solid line means a choice function ($F(r, x^{(k)}, x) = \min(f(x), w(r, x^{(k)}, x))$, where $x \in [0, 4]$ and $x^{(k)} = 1.275$); the points $x^{(k+1)}$ are the candidates for the local minimizer in the next phase.

An auxiliary minimization problem concerned with point $x^{(k)}$ and the choice function defined above is:

$$\min_{x\in\Omega} F(r, x^{(k)}, x). \tag{P$_\mathrm{k}$}$$

2.2. Properties of choice function

Firstly, a property of the cut-peak function $w(r, x^{(k)}, x)$ can be derived from Definition 2.1.1 directly.