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Adomian's decomposition method and homotopy perturbation method in solving nonlinear equations

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ABSTRACT

The Adomian's decomposition method and the homotopy perturbation method are two powerful methods which consider the approximate solution of a nonlinear equation as an infinite series usually converging to the accurate solution. By theoretical analysis of the two methods, we show, in the present paper, that the two methods are equivalent in solving nonlinear equations.

1. Introduction

One of the most basic problems in numerical analysis (and of the oldest numerical approximation problems) is that of finding the solution of the equation

$$f(x) = 0 \tag{1}$$

for a given function f which is sufficiently smooth in the neighborhood of a simple root α. In most cases it is difficult to obtain an analytical solution of (1). Therefore the exploitation of numerical techniques for solving such equations becomes a main subject of considerable interests. Probably the most well-known and widely used algorithm to find a root α is Newton's method (see e.g., [23,29,31,33]). In recent years, there have been some developments in the study of Newton-like iterative methods. To obtain these iterative methods, the Adomian's decomposition method (ADM) [7,8], the homotopy perturbation method (HPM) [17,18,22] as well as the other more general methods such as the homotopy analysis method [24,25] play an important role in the process of numerical approximation. Both ADM and HPM are the methods which consider the approximate solution of a nonlinear equation as an infinite series usually converging to the accurate solution. Over the past few years, the two methods – ADM and HPM – have been applied to solve a wide range of problems, both deterministic and stochastic, linear and nonlinear, arising from physics, chemistry, biology, engineering, etc.

At the beginning of the 80s, a new method later called ADM for solving various kinds of nonlinear equations had been proposed by Adomian [6–9]. The convergence of Adomian's method has been investigated by several authors (see e.g., [1–3, 10–12,15]). The modified ADM and its applications to the other equations have also been given, see e.g., [4,5,13,28] and references cited therein.

In recent years, some applications of the perturbation techniques [14,27] in nonlinear problems have been studied by scientists and engineers. Most perturbation methods are based on the assumption that a small parameter exists, which is too strict to find wide application. Therefore, many new techniques have been proposed to eliminate the "small parameter" assumption, such as the homotopy perturbation method [17,18,21,22]. The HPM transforms a difficult problem, under

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examination, into a simple problem which is easy to solve. Such methods have been applied to solve various nonlinear problems, see e.g., [5,13,16,20] and references cited therein.

The comparison between the ADM and HPM methods, the homotopy analysis method (HAM) and HPM, the Taylor series method and ADM, have been given through theoretical analysis and numerical analysis, see e.g., [5,19,26,30,32,34] and most other papers where the ADM and HPM methods are applied. For example, by considering the numerical solutions of a nonsingular Fredholm integral equation, Abbasbandy [5, Theorem 2.1] compared the ADM and HPM methods, and illustrated that the first method is only a special case of the second method.

In the present paper, by theoretical analysis of the ADM and HPM methods, we show that the two methods are equivalent in solving nonlinear equations such as (1). We organize the paper as follows. The basic ideas of the ADM and HPM methods are reviewed in Sections 2 and 3 respectively. The equivalence of the two methods is proved in Section 4. Finally we give a concluding remark on the applications of the two methods.

2. Adomian's decomposition method (ADM)

Let us consider the nonlinear equation (1) which can be written as the following canonical form

$$x = c + N(x), \quad (x \in \mathbb{R}) \tag{2}$$

where N is a nonlinear function and c is a constant.

The ADM method consists of representing the solution of (2) as a series

$$x = \sum_{n=0}^{\infty} x_n \tag{3}$$

and the nonlinear function as the decomposed form

$$N(x) = \sum_{n=0}^{\infty} A_n, \tag{4}$$

where A_n $(n = 0, 1, 2, \ldots)$ are the Adomian polynomials of $x_0, x_1, \ldots, x_n$ given by

$$A_n = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i x_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots. \tag{5}$$

Upon substituting (3) and (4) into (2) yields

$$\sum_{n=0}^{\infty} x_n = c + \sum_{n=0}^{\infty} A_n. \tag{6}$$

The convergence of the series in (6) gives the desired relation

$$\begin{cases} x_0 = c, \\ x_{n+1} = A_n, \quad n = 0, 1, 2, \ldots. \end{cases} \tag{7}$$

The polynomials A_n are generated for all kind of nonlinearity by Wazwaz [35]. The first few polynomials are given by

$$\begin{aligned} A_0 &= N(x_0), \\ A_1 &= x_1 N'(x_0), \\ A_2 &= x_2 N'(x_0) + \frac{1}{2}x_1^2 N''(x_0). \end{aligned} \tag{8}$$

It should be pointed out that A_0 depends only on x_0, A_1 depends only on x_0 and x_1, A_2 depends only on x_0, x_1 and x_2, and so on. Hence we may also write A_n as $A_n(x_0, x_1, \ldots, x_n)$.

Let $S_m = x_0 + x_1 + x_2 + \cdots + x_m$. Then $S_m = c + A_0 + A_1 + A_2 + \cdots + A_{m-1}$ is the $(m+1)-$term approximation of x. Such S_m can serve as a practical solution in each iteration.

3. Homotopy perturbation method (HPM)

We still consider the nonlinear Eq. (1) and recall the basic idea of the HPM (see [18, Section 2]). The basic idea of the method HPM is to construct a homotopy $H(v, p) : \mathbb{R} \times [0, 1] \longrightarrow \mathbb{R}$ which satisfies

$$H(v, p) = pf(v) + (1 - p)(f(v) - f(x_0)) = 0, \quad v \in \mathbb{R},$$

or

$$H(v, p) = f(v) - f(x_0) + pf(x_0) = 0, \quad v \in \mathbb{R}, \tag{9}$$