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Positive solutions for semi-positone three-point boundary value problems

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ABSTRACT

Using a fixed point theorem of generalized cone expansion and compression we present in this paper criteria which guarantee the existence of at least two positive solutions for semi-positone three-point boundary value problems with parameter $\lambda > 0$ belonging to a certain interval.

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1. Introduction

In this paper we consider the following second-order differential equation

$$u''(t) + \lambda f(t, u(t)) = 0, \quad t \in (0, 1), \tag{1.1}$$

subject to three-point boundary conditions

$$u(0) = 0, \qquad \alpha u(\eta) = u(1), \tag{1.2}$$

where $0 < \eta, \alpha\eta < 1$ and $f : [0, 1] \times [0, \infty) \to R$ is continuous.

Three-point boundary value problems for differential equations or difference equations arise in a variety of different areas of applied mathematics and physics. The study of multipoint boundary value problems for linear second-order ordinary differential equations was initiated in [8,9]. Motivated by the study of Il'in and Moiseev, Gupta [6] studied certain three-point boundary value problems for nonlinear ordinary differential equations. Since then, more general nonlinear three-point boundary value problems have been studied by many authors with much of the attention given to positive solutions. For a small sample of such work, we refer the reader to works [7,11,12,17]. The results of these papers are based on the Leray–Schauder continuation theorem, the nonlinear alternative of Leray–Schauder, the coincidence degree theory of Mawhin, Krasnoselskii's fixed point theorem, Schauder fixed point theorem, fixed point theorems in cones and so on. But, in most of the existing works, in order to guarantee that the operator generated by f is a cone mapping, a key condition is that the nonlinear term f must be a nonnegative function. However, the nonlinearities of many boundary value problems which

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arise in applications are not always nonnegative. When the nonlinearity may take on negative values, these problems are called semi-positone problems, which arise naturally in chemical reactor theory, design of suspension bridges, combustion and management of natural resources, see [1–3,5].

In recent papers [4,10,13–16,19], the authors considered the existence of positive solutions for semi-positone two-point, three-point or m-point boundary value problems by using Krasnoselskii's fixed point theory, Leray–Schauder fixed point theorem or fixed point index method. However, to our knowledge, the results for semi-positone three-point boundary value problems are still very few. Different from the above works mentioned, in this paper we will use a fixed point theorem of generalized cone expansion and compression to show the existence of at least two positive solutions for problem (1.1) and (1.2).

By a positive solution of (1.1) and (1.2) we understand a function $u(t)$ which is positive on $0 < t < 1$ and satisfies differential equation (1.1) and boundary conditions (1.2).

We now present a fixed point theorem of generalized cone expansion and compression which will be used in the latter proofs. Let E be a real Banach space and P be a cone in E, θ denotes the null element. The map $\rho : P \to R^1$ is said to be a convex functional on P provided that $\rho(tx + (1-t)y) \le t\rho(x) + (1-t)\rho(y)$ for all $x, y \in P$ and $t \in [0, 1]$. See [18] for further information.

Theorem 1.1 (*See [18]*). *Let Ω_1, Ω_2 be two open bounded subsets in E with $\theta \in \Omega_1, \overline{\Omega_1} \subset \Omega_2$. Suppose that $T : P \bigcap (\overline{\Omega_2} \setminus \Omega_1) \to P$ is completely continuous and $\rho : P \to [0, +\infty)$ is a uniformly continuous convex functional with $\rho(\theta) = 0$ and $\rho(x) > 0$ for $x \neq \theta$. If one of the two conditions,*

(i) $\rho(Tx) \le \rho(x), \forall x \in P \bigcap \partial\Omega_1$ *and* $\inf_{x \in P \bigcap \partial\Omega_2} \rho(x) > 0$, $\rho(Tx) \ge \rho(x), \forall x \in P \bigcap \partial\Omega_2$ *and*

(ii) $\inf_{x \in P \bigcap \partial\Omega_1} \rho(x) > 0$, $\rho(Tx) \ge \rho(x), \forall x \in P \bigcap \partial\Omega_1$ *and* $\rho(Tx) \le \rho(x), \forall x \in P \bigcap \partial\Omega_2$, *is satisfied, then T has at least one fixed point in $P \bigcap (\overline{\Omega_2} \setminus \Omega_1)$.*

The paper is organized as follows. In Section 2, we give some preliminary results that will be used in the proof of the main result. In Section 3, we prove the existence of at least two positive solutions for problem (1.1) and (1.2). In the end, we illustrate a simple use of the main result.

2. Preliminary results

In this section, we present some preliminaries which will be needed in Section 3. Consider the following boundary value problem

$$u''(t) + y(t) = 0, \quad t \in (0, 1), \tag{2.1}$$

$$u(0) = 0, \qquad \alpha u(\eta) = u(1). \tag{2.2}$$

For problem (2.1) and (2.2), we have the following conclusions which are derived from [7,11].

Lemma 2.1. *Let $\alpha\eta \neq 1$, then for $y \in C[0, 1]$, problem (2.1) and (2.2) has a unique solution*

$$u(t) = -\int_0^t (t-s)y(s)\mathrm{d}s - \frac{\alpha t}{1-\alpha\eta}\int_0^\eta (\eta - s)y(s)\mathrm{d}s + \frac{t}{1-\alpha\eta}\int_0^1 (1-s)y(s)\mathrm{d}s.$$

Remark 2.1. For the unique solution u of problem (2.1) and (2.2), we can obtain:

(i) for any $t \in [0, \eta]$,

$$\begin{aligned} u(t) &= -\int_0^t (t-s)y(s)\mathrm{d}s - \frac{\alpha t}{1-\alpha\eta}\int_0^\eta (\eta - s)y(s)\mathrm{d}s + \frac{t}{1-\alpha\eta}\int_0^1 (1-s)y(s)\mathrm{d}s \\ &= \frac{t}{1-\alpha\eta}\int_\eta^1 (1-s)y(s)\mathrm{d}s + \frac{t}{1-\alpha\eta}\int_t^\eta (1 - s - \alpha\eta + \alpha s)y(s)\mathrm{d}s \\ &\quad + \frac{1}{1-\alpha\eta}\int_0^t (s - ts + \alpha ts - \alpha s\eta)y(s)\mathrm{d}s, \end{aligned}$$

(ii) for any $t \in (\eta, 1]$,

$$\begin{aligned} u(t) &= -\int_0^t (t-s)y(s)\mathrm{d}s - \frac{\alpha t}{1-\alpha\eta}\int_0^\eta (\eta - s)y(s)\mathrm{d}s + \frac{t}{1-\alpha\eta}\int_0^1 (1-s)y(s)\mathrm{d}s \\ &= \frac{t}{1-\alpha\eta}\int_t^1 (1-s)y(s)\mathrm{d}s + \frac{1}{1-\alpha\eta}\int_\eta^t (s - st + \alpha\eta t - \alpha\eta s)y(s)\mathrm{d}s \\ &\quad + \frac{1}{1-\alpha\eta}\int_0^\eta (s - st + s\alpha t - s\alpha\eta)y(s)\mathrm{d}s. \end{aligned}$$

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