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Erratum

Erratum to: "A note on the preconditioned Gauss–Seidel method for M-matrices" [J. Comput. Appl. Math. 219 (1) (2008) 59–71][☆]

Qingbing Liu [a,b], Guoliang Chen [a,*]

[a] Department of Mathematics, East China Normal University, Shanghai 200062, PR China

[b] Department of Mathematics, Zhejiang Wanli University, Ningbo 315100, PR China

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ABSTRACT

In this note, some errors in a recent article by Niki et al. [H. Niki, T. Kohno, M. Morimoto, The preconditioned Gauss–Seidel method faster than the *SOR* method, J. Comput. Appl. Math. 219 (2008) 59–71] are pointed out and a new proof for the corresponding result is presented.

1. Introduction

In this note, we consider the following linear system

$$Ax = b \quad x, b \in R^n, \tag{1}$$

where $A = (a_{ij}) \in R^{n\times n}$ is an M-matrix. The basic iterative method for solving the linear system (1) can be expressed as

$$x_{k+1} = M^{-1}Nx_k + M^{-1}b \quad k = 0, 1, \ldots, \tag{2}$$

where x_0 is an initial vector and $A = M - N$ is a splitting of A. The $M^{-1}N$ is called an iteration matrix of the basic iterative method.

To improve the convergence rate of the basic iterative method, several preconditioned iterative methods have been proposed [1–5]. The main idea of these preconditioned iterative methods is to transform the original system into the preconditioned form

$$PAx = Pb, \tag{3}$$

where $P \in R^{n\times n}$ is a nonsingular preconditioner. Then the corresponding basic iterative method is given in general by

$$x_{k+1} = M_p^{-1}N_px_k + M_p^{-1}Pb \quad k = 0, 1, \ldots, \tag{4}$$

where $PA = M_p - N_p$ is a splitting of PA and M_p is nonsingular, similarly to the original system (1). We call the basic iterative method corresponding to the preconditioned system the preconditioned iterative method, such as the preconditioned

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* Corresponding author.
E-mail address: glchen@math.ecnu.edu.cn (G. Chen).

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Gauss–Seidel. Under certain assumptions, it has been proved that the preconditioned Gauss–Seidel method is superior to the basic Gauss–Seidel method and the *SOR* method.

The preconditioner P was introduced in [1] as follows:

$$P = I + S, \tag{5}$$

where

$$S = \begin{bmatrix} 0 & -a_{12} & 0 & \cdots & 0 \\ 0 & 0 & -a_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -a_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{6}$$

Kohno et al. [2] proposed using preconditioner (5) with some parameters, i.e.,

$$P_\alpha = I + D(\alpha)S \tag{7}$$

instead of $P = I + S$, where $D(\alpha) = \mathrm{diag}(\alpha_1, \ldots, \alpha_{n-1}, 1)$.

In [9] the authors considered the preconditioner with $P = I + R$ and $P = I + S + R$, where S is given in (6) and

$$R = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ -a_{n1} & \cdots & -a_{n,n-1} & 0 \end{bmatrix}. \tag{8}$$

Several results in [9] are given below.

Theorem 1.1 (*Niki et al. [9, Theorem 2.9]*). *Let A be an M-matrix, and both $A = M - N$, $(I+R)A = M_{R_1} - N_{R_1}$ be the Gauss–Seidel convergent splittings. Then the following inequality holds:*

$$\rho(M_{R_1}^{-1}N_{R_1}) \leq \rho(M^{-1}N) < 1.$$

Theorem 1.2 (*Niki et al.[9, Theorem 2.10]*). *Let A be an M-matrix. Assume that $a_{nj} \leq a^R_{nj}/a^R_{nn}$, for $1 \leq j \leq n-1$. Then $A_S = (I + S)A = M_S - N_S$ and $A_R = (I + S + R)A = M_R - N_R$ are Gauss–Seidel splittings. Then the following inequality holds.*

$$\rho(M_R^{-1}N_R) \leq \rho(M_S^{-1}N_S).$$

Theorem 1.3 (*Niki et al.[9, Theorem 2.11]*). *Let $A = I - L - U$ be a nonsingular M-matrix. Assume that $a_{nj} \neq 0, j = 1, \ldots, n$ and*

$$u^t \leq v_n^{-1} v^t. \tag{9}$$

Then

$$\rho(T_R) \leq \rho(T_S) < \rho(T) < 1.$$

where $u^t = (a_{n1}, \ldots, a_{n,n-1})$, $v^t = (v_1, \ldots, v_{n-1})$,

$$v_j = a^R_{nj} = \begin{cases} a_{nj} - \displaystyle\sum_{k=1,k\neq j}^{n-1} a_{nk}a_{kj}, & 1 \leq j \leq n-1, \\ 1 - \displaystyle\sum_{k=1}^{n-1} a_{nk}a_{kn}, & j = n. \end{cases}$$

This note is organized as follows. In Section 2, we present some definitions and preliminary results, and give two counterexamples to illustrate that there are some errors in the proofs of Theorems 1.1 and 1.2. In Section 3, a correct proof of Theorem 1.1 is presented. We show that a correct proof of Theorem 1.2 is given in [8].

2. Preliminaries

We first recall the following: A matrix $A = (a_{ij}) \in R^{n\times n}$ is called a Z-matrix if $a_{ij} \leq 0$ for $i \neq j$. A real vector $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ is called nonnegative(positive) and denoted by $x \geq 0 (x > 0)$ if $x_i \geq 0 (x_i > 0)$ for all i. Similarly,