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A new criterion for global robust stability of interval delayed neural networks

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Abstract

A novel criterion for the global robust stability of Hopfield-type interval neural networks with delay is presented. An example showing the effectiveness of the present criterion is given.

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1. Introduction

This paper deals with the delayed neural network model defined by the following state equations:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \boldsymbol{u} \tag{1}$$

or

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau)) + u_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^{\mathrm{T}}$ is the state vector associated with the neurons, $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix ($c_i > 0, i = 1, 2, \ldots, n$), $\boldsymbol{A} = (a_{ij})_{n\times n}$ and $\boldsymbol{B} = (b_{ij})_{n\times n}$ are the connection weight and the delayed connection weight matrices, respectively, $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^{\mathrm{T}}$ is a constant external input vector, τ is the transmission delay, the f_j, $j = 1, 2, \ldots, n$, are the activation functions, $\boldsymbol{f}(\boldsymbol{x}(\cdot)) = \begin{bmatrix} f_1(x_1(\cdot)) & f_2(x_2(\cdot)) & \cdots & f_n(x_n(\cdot)) \end{bmatrix}^{\mathrm{T}}$, and the superscript ‘T’ to any vector (or matrix) denotes the transpose of that vector (or matrix). The activation functions are assumed to satisfy the following restrictions:

$$\left| f_j(\xi) \right| \le M_j \quad \forall \xi \in R;\; M_j > 0,\; j = 1, 2, \ldots, n \tag{3}$$

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and

$$0 \le \frac{f_j(\xi_1) - f_j(\xi_2)}{\xi_1 - \xi_2} \le L_j \quad j = 1, 2, \ldots, n \tag{4}$$

for each $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$, where L_j are positive constants. Such activation functions ensure the existence of an equilibrium point for Eq. (1) [1]. The quantities c_i, a_{ij}, and b_{ij} may be considered as intervalized as follows:

$$\boldsymbol{C}_I := [\underline{\boldsymbol{C}}, \overline{\boldsymbol{C}}] = \left\{\boldsymbol{C} = \mathrm{diag}(c_i) : \underline{\boldsymbol{C}} \le \boldsymbol{C} \le \overline{\boldsymbol{C}}, \text{ i.e., } \underline{c}_i \le c_i \le \bar{c}_i,\ i = 1, 2, \ldots, n\right\} \tag{5}$$

$$\boldsymbol{A}_I := [\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \underline{\boldsymbol{A}} \le \boldsymbol{A} \le \overline{\boldsymbol{A}}, \text{ i.e., } \underline{a}_{ij} \le a_{ij} \le \bar{a}_{ij}, i, j = 1, 2, \ldots, n\right\} \tag{6}$$

$$\boldsymbol{B}_I := [\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}] = \left\{\boldsymbol{B} = (b_{ij})_{n\times n} : \underline{\boldsymbol{B}} \le \boldsymbol{B} \le \overline{\boldsymbol{B}}, \text{ i.e., } \underline{b}_{ij} \le b_{ij} \le \bar{b}_{ij}, i, j = 1, 2, \ldots, n\right\}. \tag{7}$$

Definition. The system given by Eq. (1) with the parameter ranges defined by Eqs. (5)–(7) is globally robustly stable if the unique equilibrium point $\boldsymbol{x}^* = \begin{bmatrix} x_1^* & x_2^* & \cdots & x_n^* \end{bmatrix}^{\mathrm{T}}$ of the system is globally asymptotically stable for all $\boldsymbol{C} \in \boldsymbol{C}_I, \boldsymbol{A} \in \boldsymbol{A}_I, \boldsymbol{B} \in \boldsymbol{B}_I$.

In the following, $\boldsymbol{F} > \boldsymbol{0}$ denotes that the matrix $\boldsymbol{F}$ is symmetric positive definite. If $\boldsymbol{W}$ is a matrix, its norm $\|\boldsymbol{W}\|_2$ is defined as $\|\boldsymbol{W}\|_2 = \sup\{\, \|\boldsymbol{W}\boldsymbol{x}\| : \|\boldsymbol{x}\| = 1\} = \sqrt{\lambda_{\max}(\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W})}$, where $\lambda_{\max}(\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W})$ denotes the maximum eigenvalue of $\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}$.

The problem of global asymptotic stability of Eq. (1) has generated considerable interest. For a sample of literature on the subject, the reader is referred to [1–28] and the references cited therein. The problem of global robust stability of Eq. (1) with the intervalized parameters given by Eqs. (5)–(7) has also received considerable attention [1–13]. In this paper, we present a novel criterion for the global robust stability of Eq. (1) with Eqs. (5)–(7). An example showing the effectiveness of the present criterion is given.

2. The criterion

Define

$$\boldsymbol{A}^* = \left(a_{ij}^*\right)_{n\times n} = (\overline{\boldsymbol{A}} + \underline{\boldsymbol{A}})/2, \qquad \boldsymbol{A}_* = \left(a_{*ij}\right)_{n\times n} = (\overline{\boldsymbol{A}} - \underline{\boldsymbol{A}})/2. \tag{8}$$

Let the interval given by Eq. (6) be divided into the following two equal intervals:

$$\boldsymbol{A}_I^{\mathrm{I}} := [\underline{\boldsymbol{A}}, \boldsymbol{A}^*] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \underline{\boldsymbol{A}} \le \boldsymbol{A} \le \boldsymbol{A}^*, \text{ i.e., } \underline{a}_{ij} \le a_{ij} \le a_{ij}^*, i, j = 1, 2, \ldots, n\right\} \tag{9}$$

$$\boldsymbol{A}_I^{\mathrm{II}} := [\boldsymbol{A}^*, \overline{\boldsymbol{A}}] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \boldsymbol{A}^* \le \boldsymbol{A} \le \overline{\boldsymbol{A}}, \text{ i.e., } a_{ij}^* \le a_{ij} \le \overline{a}_{ij}, i, j = 1, 2, \ldots, n\right\}. \tag{10}$$

Define the symmetric matrix $\boldsymbol{S}^{\mathrm{I}} = \left\{s_{ij}^{\mathrm{I}}\right\}_{n\times n}$ as

$$s_{ij}^{\mathrm{I}} = \begin{Bmatrix} -2a_{ii}^*, & \text{if } i = j \\ -\hat{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \quad \hat{a}_{ij} = \max\left\{ \left|a_{ij}^* + a_{ji}^*\right|, \left|\underline{a}_{ij} + \underline{a}_{ji}\right|\right\} \tag{11}$$

and the symmetric matrix $\boldsymbol{S}^{\mathrm{II}} = \left\{s_{ij}^{\mathrm{II}}\right\}_{n\times n}$ as

$$s_{ij}^{\mathrm{II}} = \begin{Bmatrix} -2\overline{a}_{ii}, & \text{if } i = j \\ -\tilde{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \quad \tilde{a}_{ij} = \max\left\{ \left|\overline{a}_{ij} + \overline{a}_{ji}\right|, \left|a_{ij}^* + a_{ji}^*\right|\right\}. \tag{12}$$

The main result is given in the following theorem.