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On the relationships between G-preinvex functions and semistrictly G-preinvex functions☆

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Abstract

A new class of functions, termed semistrictly G-preinvex functions, is introduced in this paper. The relationships between semistrictly G-preinvex functions and G-preinvex functions are investigated under mild assumptions. Our results improve and extend the existing ones in the literature.

MSC: 90C26

Keywords: G-preinvex functions; Semistrictly G-preinvex functions; Invex set; Continuity

1. Introduction

Convexity and some generalizations of convexity play a crucial role in mathematical economics, engineering, management science, and optimization theory. Therefore, to consider wider and wider classes of generalized convex functions is important, but it is also important to seek practical criteria for convexity or generalized convexity (see [1–10] and the references therein). Two significant generalizations of convex functions are the so-called preinvex functions introduced in [7] and prequasi-invex functions given in [6]. Yang et al. [10] established characterizations of prequasi-invex functions under a semicontinuity condition; Luo et al. in [4,5] improved their results in [10] under much weaker assumptions. Yang and Li [8] presented some properties of preinvex functions; they in [9] introduced the semistrictly preinvex function and established relationships between preinvex functions and semistrictly preinvex functions under a certain set of conditions. Recently, Antczak [2,3] introduced the concept of the G-preinvex function, which includes the preinvex function [7] and r-preinvex function [1] as special cases. Relations of this G-preinvex function to preinvex functions and some properties of this class of functions were studied in [2].

In this paper, we introduce a new class of functions called semistrictly G-preinvex functions, which include semistrictly preinvex functions [9] as a special case. We investigate the relationship between G-preinvex functions

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and semistrictly G-preinvex functions under mild conditions. It is worth pointing out that the results obtained here improve and generalize the corresponding ones given in [9].

The rest of the paper is organized as follows. In Section 2, we give some preliminaries. The main results of the paper are presented in Section 3, and their proofs are given in Appendix. Section 4 gives some conclusions.

2. Preliminaries

In this setion, we will describe some definitions of generalized convexity.

Definition 1 (*[7]*). For a given set $K \subseteq \mathbb{R}^n$ and a given function $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, K is said to be an invex set with respect to η iff

$$\forall x, y \in K,\ \forall \lambda \in [0, 1] \Rightarrow y + \lambda\eta(x, y) \in K.$$

Definition 2 (*[7]*). Let $K \subseteq \mathbb{R}^n$ be an invex set with respect to $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$. The function $f : K \to \mathbb{R}$ is said to be preinvex on K iff, $\forall x, y \in K,\ \forall \lambda \in [0, 1]$,

$$f(y + \lambda\eta(x, y)) \le \lambda f(x) + (1 - \lambda) f(y).$$

Definition 3 (*[9]*). Let $K \subseteq \mathbb{R}^n$ be an invex set with respect to $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$. The function $f : K \to \mathbb{R}$ is said to be semistrictly preinvex on K iff, $\forall x, y \in K,\ f(x) \neq f(y),\ \forall \lambda \in (0, 1)$,

$$f(y + \lambda\eta(x, y)) < \lambda f(x) + (1 - \lambda) f(y).$$

In [9], the relationship between preinvex functions and semistrictly preinvex functions was discussed under the following condition.

Condition C (*[7]*). Let $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$. We say that the function η satisfies Condition C iff, $\forall x, y \in K, \lambda \in [0, 1]$,

$$\eta(y, y + \lambda\eta(x, y)) = -\lambda\eta(x, y),$$
$$\eta(x, y + \lambda\eta(x, y)) = (1 - \lambda)\eta(x, y).$$

Let $I_f(K)$ be the range of f, i.e., the image of K under f, and G^{-1} be the inverse of G.

Definition 4 (*[2,3]*). Let $K \subseteq \mathbb{R}^n$ be an invex set (with respect to η). The function $f : K \to \mathbb{R}$ is said to be G-preinvex on K iff there exist a continuous real-valued increasing function $G : I_f(K) \to \mathbb{R}$ and a vector-valued function $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ such that, $\forall x, y \in K,\ \forall \lambda \in [0, 1]$,

$$f(y + \lambda\eta(x, y)) \le G^{-1}(\lambda G(f(x)) + (1 - \lambda)G(f(y))).$$

We note that the G-preinvex function in Definition 4 reduces to the preinvex function in Definition 2 and the r-preinvex function in [1] when setting $G(t) = t$ and $G(t) = \mathrm{e}^{rt}$, respectively.

We now introduce a new kind of generalized convex function termed semistrictly G-preinvex function as follows.

Definition 5. Let $K \subseteq \mathbb{R}^n$ be an invex set (with respect to η). The function $f : K \to \mathbb{R}$ is said to be semistrictly G-preinvex on K iff there exist a continuous real-valued increasing function $G : I_f(K) \to \mathbb{R}$ and a vector-valued function $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ such that, $\forall x, y \in K,\ f(x) \neq f(y),\ \forall \lambda \in (0, 1)$,

$$f(y + \lambda\eta(x, y)) < G^{-1}(\lambda G(f(x)) + (1 - \lambda)G(f(y))).$$

We also observe that the semistrictly G-preinvex function in Definition 5 is a generalization of the semistrictly preinvex function in Definition 3 when taking $G(t) = t$.